Non-Perturbative Constraints on Light Sparticles from Properties of the RG Flow

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Matthew Buican

Rutgers University

M. B., 1206.3033 [hep-th]; 1109.3279 [hep-th], PRD;

S. Abel, M. B., and Z. Komargodski; 1105.2885 [hep-th], PRD.

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Overview

• General properties of the 4D RG flow.

• R-symmetric RG flows: particle physics motivation and canonical non-perturbative operator multiplets (R_μ, U).

• Canonical soft masses.

• Emergent SUSY and light sparticles.

• Universality of emergent SUSY and the flow of soft masses.

• Model building directions.

RG Generalities

• Under rather general assumptions, UV-complete QFTs can be understood as interpolations between UV and IR scale-invariant limits (may also be gapped and hence empty in IR).

• Given well-defined operators and correlation functions of the UV theory, can we say something about the corresponding objects in the IR?

• What are the emergent symmetries of the IR fixed points? What are the broken symmetries?

• In general, new internal and space-time symmetries. What are they? How do we get a handle on them?

RG Generalities (cont...)

• Non-perturbative dynamics along the RG flow make these questions hard to answer. Although, we do have powerful tools like the a-theorem [Cardy, '88], [Komargodski and Schwimmer, '11], [Komargodski, '11], [Intriligator and Wecht, '03], [Anselmi et. al.], [Kutasov et. al.].

• We will specialize to four-dimensional R-symmetric theories.

• As we will see SUSY, and, in particular, R-symmetry give us strong handles to use to answer a lot of these questions in controlled settings. Additional tools that compare UV and IR behavior beyond the a-theorem [M. B., '11]?

• Furthermore, studying such RG flows may lead to interesting applications to particle physics.

The R-Symmetric RG Flow and Particle Physics

• LHC $\Rightarrow$ light Higgs $\Rightarrow$ SUSY?

• If SUSY exists, then it is broken, and need hidden sector. Should have some type of (approximate) R-symmetry [Nelson and Seiberg, '93], [Intriligator, Seiberg, and Shih, '07].

• If SUSY is broken dynamically, there will be some type of strong coupling involved $\Rightarrow$ study general non-perturbative aspects of R-symmetric theories.

• Also, we frequently have some emergent bosonic symmetries in such theories (ISS, etc.) $\Rightarrow$ Constraints on emergent symmetries should lead to constraints on DSB.

Particle physics motivation (cont...)

• ... But sparticles still haven't been observed. If SUSY is to remain "natural," we need light stops. This suggests a sector in which SUSY breaking is suppressed.

• We will suggest a new non-perturbative RG rule (an inequality in the spirit of the a-theorem) applicable to a broad class of R-symmetric theories (with and without holographic descriptions).

• This rule will be related to the emergence of accidental symmetries (both bosonic and fermionic) in the IR and will constrain the emergence of light sparticles.

Canonical Non-Perturbative D.O.F.'s

• Our theories have a conserved R-current at all energy scales.

• Since $[R, Q] \sim Q$, $\{Q, \bar{Q}\} \sim P$, the R-current transforms in a multiplet with $S_{\mu\alpha}$ and $T_{\mu\nu}$.

$$\bar{D}^{\dot{\alpha}} \mathcal{R}_{\dot{\alpha}\alpha} = \chi_\alpha \ , \quad D^\alpha \chi_\alpha - \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \chi_\alpha = 0 \ . \qquad (1)$$

• For $\chi_\alpha \neq 0$, not the usual Ferrara-Zumino (FZ) supercurrent multiplet. Note that χ_α contains the trace anomaly.

• When $\chi_\alpha = 0$, this is the superconformal R-symmetry.

• There is an ambiguity in the above equation under $\mathcal{R}_{\alpha\dot{\alpha}} \rightarrow \mathcal{R}_{\alpha\dot{\alpha}} + [D_\alpha, \bar{D}_{\dot{\alpha}}] \, J$ and $\chi_\alpha \rightarrow \chi_\alpha + \frac{3}{2} \bar{D}^2 D_\alpha J$ for conserved J, i.e., $\bar{D}^2 J = 0$. This affects the supercurrent and stress tensor through improvements.

Canonical Non-Perturbative D.O.F.'s (cont...)

• For the theories we will consider (those that also have an FZ multiplet), can write [Komargodski and Seiberg, '10]

$$\chi_\alpha = \bar{D}^2 D_\alpha U \ , \tag{2}$$

for a well-defined (and away from the endpoints of the RG flow, **non-conserved**) U. U contains the **trace anomaly**.

• Solving the above equations in the UV of an asymptotically free theory, we find

$$\begin{aligned} \mathcal{R}^{UV}_{\alpha\dot{\alpha}} &= \sum_i \left(2 D_\alpha \Phi_i \bar{D}_{\dot{\alpha}} \bar{\Phi}^i - r_i [D_\alpha, \bar{D}_{\dot{\alpha}}] \Phi_i \bar{\Phi}^i \right) , \\ U^{UV} &= -\frac{3}{2} \sum_i \left(r_i - \frac{2}{3} \right) \bar{\Phi}^i \Phi_i \ . \end{aligned} \tag{3}$$

More generally: $U^{UV}_\mu = \frac{3}{2}\left(R^{UV}_\mu - \tilde{R}^{UV}_\mu \right)$.

The R-symmetry Current and the RG Flow

• **Idea:** Study $\mathcal{R}$, U along RG. Conserved $\mathcal{R}$ gives handle on U.

• **Assumption**: UV and IR fixed points are SCFTs (can be made rigorous in "SQCD-like" theories [`I. Antoniadis and M.B., '11`])

• At the IR fixed point, we know what should happen to $\mathcal{R}_{\alpha\dot{\alpha}}$. Indeed, either this multiplet flows to the superconformal R-multiplet or to an object that can be improved to the superconformal R-multiplet:

$$\tilde{\mathcal{R}}_{\alpha\dot{\alpha}} = \mathcal{R}^{IR}_{\alpha\dot{\alpha}} - [D_\alpha, \bar{D}_{\dot{\alpha}}]J \ , \quad \tilde{U} = U^{IR} - \frac{3}{2}J = 0 \ . \tag{4}$$

Determine $\tilde{\mathcal{R}}_{\alpha\dot{\alpha}}$ from duality or a-maximization.

• **Upshot:** Therefore, $U \to \frac{3}{2}J$, where $U^{IR}_\mu = \frac{3}{2}\left(R^{IR}_\mu - \tilde{R}^{IR}_\mu\right)$.

The R-symmetry Current and the RG Flow (cont...)

• J may be a conserved current of the full theory or an accidental symmetry of the IR. We will see an extreme version of this for SQCD in the free magnetic range.

• In the case of a free magnetic phase, we have

$$U^{IR} = -\frac{3}{2}\sum_i \left(r_i - \frac{2}{3}\right)\bar{\phi}^i\phi_i \ , \qquad (5)$$

for the "emergent" d.o.f.'s.

Toward a Unique (R_μ, U) Pair

• As we discussed above, theories with global symmetries contain an infinite family of R-currents. If we want to make general statements about the RG flow, which one do we pick?

• We want one sensitive to accidental symmetries.

• **Natural candidate:** Up to caveats (see [M.B. '11] and [M.B. '12]) (R_μ, U) defined by performing a-maximization in the deformed UV theory.

Toward a Unique (R_μ, U) Pair (cont...)

• We start by using a-maximization to find the UV superconformal R-current; consider $\mathcal{R}^{t*}_{\mu,UV} = \mathcal{R}^{(0)*}_{\mu,UV} + \sum_i t^i J^{UV*}_{\mu,i}$, where $J^{UV*}_{\mu,i}$ are the full set of non-R symmetries of the UV SCFT.

• Taking $\tilde{a}^t_{UV} = 3\mathrm{Tr}\ \left(\mathcal{R}^{t*}_{UV}\right)^3 - \mathrm{Tr}\ \mathcal{R}^{t*}_{UV}$, solve $\partial_{t^i}\tilde{a}^t_{UV}|_{t^i=t^i_*} = 0$, $\quad \partial^2_{t^i t^j}\tilde{a}^t_{UV}|_{t^{i,j}=t^{i,j}_*} < 0$. This defines $\tilde{R}^{UV}_\mu$.

• Deform the theory by turning on an R-symmetry-preserving relevant deformation and/or an R-symmetry-preserving vev. Now only $\left\{\hat{J}^{UV*}_{\mu,a}\right\} \subset \left\{J^{UV*}_{\mu,i}\right\}$ are still conserved currents that respect the vacuum.

Toward a Unique (R_μ, U) Pair (cont...)

• Maximizing $\tilde{a}$ over this subset yields $\mathcal{R}_\mu^{UV} = \mathcal{R}_\mu^{(0),UV} + \sum_a \hat{t}_*^a \hat{J}_{\mu,a}^{UV}$ and U^{UV} which descend from a corresponding pair in the undeformed UV SCFT, $(\mathcal{R}_{\mu,\text{vis}}^{UV}, U_{\text{vis}}^{UV})$. See [M.B., '11] and [M.B., '12] for a slightly more general definition (including a way to take into account massive currents).

Properties of the U_{vis} Multiplet

• The two point function $\langle U_{\mu\text{vis}}(x) U_{\text{vis}}(0)\rangle$ is given by one overall coefficient in the UV and one in the IR, τ_U^{IR} and τ_U^{UV}

$$\langle U_{\mu,\text{vis}}^{UV,IR}(x) U_{\nu,\text{vis}}^{UV,IR}(0)\rangle = \frac{\tau_U^{UV,IR}}{(2\pi)^4}\left(\partial^2\eta_{\mu\nu} - \partial_\mu\partial_\nu\right)\frac{1}{x^4}\ . \qquad (6)$$

We work modulo chiral plus anti-chiral terms.

• In a large variety of strongly coupled (and weakly coupled) theories with dramatically different dynamics (interacting fixed points, confinement with and without chiral symmetry breaking, etc.) we find [M.B., '11]

$$\tau_U^{UV} > \tau_U^{IR}\ . \qquad (7)$$

Properties of the U_{vis} Multiplet (cont...)

• In this talk, we will conjecture (but not prove) that $\tau_U^{UV} > \tau_U^{IR}$ is true in every R-symmetric theory with an FZ multiplet.

• This law implies that the mixing of accidental symmetries with the IR superconformal R-current is bounded above by τ_U^{UV}.

• This law correctly predicts non-trivial phase transitions in the IR of SQCD and also constrains the theory of Intriligator, Seiberg, and Shenker, suggesting that it does not break SUSY, [M.B., '11].

Example: SQCD in the Free Magnetic Range

- Consider $SU(N_c)$ with $N_c + 1 < N_f \leq 3N_c/2$: this is a flow between Gaussian fixed points

- The UV (electric) theory:

$$
\begin{array}{ccccc}
 & SU(N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\
 & & & & \\
Q & \mathbf{N_c} & \mathbf{N_f}\times \mathbf{1} & 1-\frac{N_c}{N_f} & 1 \\
\tilde{Q} & \bar{\mathbf{N}}_\mathbf{c} & \mathbf{1}\times \bar{\mathbf{N}}_\mathbf{f} & 1-\frac{N_c}{N_f} & -1
\end{array}
\tag{8}
$$

- $U_{\text{vis}}^{UV} = -\left(\frac{1}{2} - \frac{3N_c}{2N_f}\right)\left(Q\bar{Q} + \tilde{Q}\bar{\tilde{Q}}\right)$.

Example: SQCD in the Free Magnetic Range (cont...)

- We have the following IR (magnetic) theory [Seiberg, '94]

$$\begin{array}{ccccc} & SU(N_f - N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\ q & \mathbf{N_f - N_c} & \mathbf{\bar{N}_f \times 1} & \frac{N_c}{N_f} & \frac{N_c}{N_f - N_c} \\ \tilde{q} & \mathbf{\bar{N}_f - \bar{N}_c} & \mathbf{1 \times \bar{N}_f} & \frac{N_c}{N_f} & -\frac{N_c}{N_f - N_c} \\ M & \mathbf{1} & \mathbf{N_f \times N_f} & 2 - 2\frac{N_c}{N_f} & 0 \end{array} \quad (9)$$

- $U_{\text{vis}}^{IR} = 2\left(1 - \frac{N_c}{N_f}\right) M\bar{M} + \frac{N_c}{N_f}\left(q\bar{q} + \tilde{q}\bar{\tilde{q}}\right)$.

$$\tau_U^{UV} = \frac{N_c(N_f - 3N_c)^2}{2N_f} > \frac{(3N_f - N_c)(3N_c - 2N_f)^2}{2N_f} = \tau_U^{IR} \ . \quad (10)$$

- Many strong coupling checks via anomaly matching [M.B., '11].

Canonical SUSY Breaking

• The U_{vis} multiplet provides also a canonical way to break SUSY:

$$\delta S_{UV}|_{SSB} = -\int d^4x \ \lambda \cdot U_{\text{vis}}^{UV}| \ . \tag{11}$$

• For a general strongly-coupled SCFT, the above deformation is simply a SUSY-breaking deformation by an operator of dimension two and does not admit an interpretation in terms of particles (i.e., it is not a "mass" term).

• When the theory is asymptotically free, however, recall $U_{\text{vis}}^{UV} = -\sum_i U_{\text{vis}}^{UV}(\Phi_i) \cdot \Phi_i \bar{\Phi}^i, \quad U_{\text{vis}}^{UV}(\Phi_i) = \frac{3}{2}\left(\mathcal{R}_{\text{vis}}^{UV}(\Phi_i) - \frac{2}{3}\right)$ and so

$$\delta S_{UV}|_{SSB} = -\int d^4x \ (m_i^{UV})^2 \cdot \bar{\phi}^i \phi_i, \quad (m_i^{UV})^2 \equiv m^2 \cdot U_{\text{vis}}^{UV}(\phi_i) \ . \tag{12}$$

Canonical SUSY Breaking (cont...)

- Turn on relevant SUSY deformation too (also can turn on vev)

$$\delta S_{UV} = -\int d^4x \left(\int d^2\theta \ \lambda_{\mathcal{O}} \cdot \mathcal{O} + \text{h.c.} + \lambda \cdot U_{\text{vis}}^{UV}| \right) \ , \tag{13}$$

and work in probe approximation: i.e., don't allow SUSY-breaking operator to back-react on the strong dynamics (leading-order approximation in which we take λ to be parametrically small). [Arkani-Hamed and Rattazzi, '98], [Zoupanos et. al., '98], [Luty and Ratazzi, ' 99], [Abel, Komargodski, and M.B., '11]

- Follow to deep IR and find

$$\delta S_{IR}|_{SSB} = -\int d^4x \ \lambda \cdot U_{\text{vis}}^{IR}| \ . \tag{14}$$

- $\delta\tau_U > 0 \Rightarrow$ norm of IR SUSY breaking operator smaller than UV one $\Rightarrow$ bound on emergent SUSY!

Canonical SUSY Breaking (cont...)

• When IR is free, $U^{IR}_{\text{vis}} = -\sum_a U^{IR}_{\text{vis}}(\hat{\Phi}_a) \cdot \hat{\Phi}_a \bar{\hat{\Phi}}^a$, $U^{IR}_{\text{vis}}(\hat{\Phi}_a) = \frac{3}{2}\left(\mathcal{R}^{IR}_{\text{vis}}(\hat{\Phi}_a) - \frac{2}{3}\right)$ and have

$$\delta S_{IR}|_{SSB} = -\int d^4x \, (m^{IR}_a)^2 \cdot \bar{\hat{\phi}}^a \hat{\phi}_a, \quad (m^{IR}_a)^2 \equiv m^2 \cdot U^{IR}_{\text{vis}}(\hat{\phi}_a) \; , \quad (15)$$

and so $\delta\tau_U > 0 \Rightarrow$

$$\sum_a (m^{IR}_a)^4 < \sum_i (m^{UV}_i)^4 \; , \quad (16)$$

i.e., a bound on leading-order IR scalar masses!

Example: SUSY Breaking in SQCD and Light Stops

• Since the SUSY-breaking term proportional to $U_{\text{vis}}|$ acquires anomalous dimension, the heuristic expectation is that it flows to zero at leading order. However, we see that this is **not** true in general since U_{vis} can mix with accidental symmetries in the IR (and thus flow to something of IR dimension two).

• Such examples include free-magnetic SQCD, but our formulation in terms of currents makes this simple to understand even in more general theories (like adjoint SQCD [`Abel, Komargodski, Buican, '11`] with interacting IR fixed points and accidental symmetries) that don't have known duals or sometimes lack even Lagrangian descriptions. For simplicity will discuss SQCD and make contact with phenomenology.

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• One natural theory to study is SQCD deformed by the canonical soft term described above

$$\delta S_{UV}|_{SSB} = \int d^4x\ m^2 \left(Q\bar{Q} + \tilde{Q}\bar{\tilde{Q}}\right) \ , \tag{17}$$

• Since we expect this SUSY breaking to by suppressed in the IR, can naturally imagine embedding stops (or, more accurately, their UV ancestors) in the Q, $\tilde{Q}$ fields. If the remaining two generations are singlets under the SQCD strong dynamics (the SM gauge group is just a weakly gauged subgroup of the flavor symmetry) we naturally expect the stops to be light compared to the first two generations **[Csaki, Randall, and Terning, '12]**. We see, however, that this picture is more general!

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• **Note:** There will generally be important corrections to the stop mass from weakly-coupled spectators (i.e., the rest of the SSM fields including the gauginos). However, we will assume that the duality scale is low. Compositeness solves hierarchy problem and SUSY solves little hierarchy problem with light stops.

• We will instead be concerned with the strong-coupling RG flow of the $U_{\text{vis}} = Q\bar{Q} + \tilde{Q}\bar{\tilde{Q}}$ operator. Consider first $3N_c/2 < N_f < 3N_c$. Conformal window with no accidental symmetries $\Rightarrow U_{\text{vis}} \to 0$. $U_{\text{vis}} \to 0$ for $N_f = 3N_c/2$ as well, **however**, there are accidental symmetries in this phase.

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• Although $U_{\text{vis}} \to 0$, SUSY breaking may still be important in the IR since the importance of the term $\mathcal{L}_{IR} \supset m^2 U_{IR}$ depends on *how* $U_{\text{vis}} \to 0$, i.e., as a power-law or logarithmically.

• IR behavior intimately connected to existence and mixings of accidental symmetries.

• Note: $U_{\text{vis}} \to 0$ logarithmically only if approach to IR CFT is via a marginally irrelevant operator.

• **General SCFT Fact:** An operator is marginally irrelevant only if it breaks a symmetry of the SCFT, [`Green et. al., '10`].

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• **Implication:** $U_{\text{vis}} \to 0$ logarithmically only if it mixes with an accidental symmetry away from the IR fixed point

$$U = \gamma J \ , \tag{18}$$

γ an anomalous dimension computable (even for strong coupling) in conformal perturbation theory, [Green et. al., '10] with $\gamma \to 0$ in deep IR.

• For $N_f = 3N_c/2$ (and similar parameter choices for SO and Sp gauge groups), $U_{\text{vis}} \to 0$ logarithmically because we have accidental symmetries. Can simply understand this from toy theory with $K = Z(\mu)\Phi\bar{\Phi}$ and $W = \lambda\Phi^3$. In holomorphic scheme $U_{\text{vis}} \sim dK/d\log\mu$, and so have logarithmic running with $U_{\text{vis}} = |\lambda|^2\Phi\bar{\Phi}$. Illustration of the power of currents!

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• On the other hand, inside conformal window $U_{\text{vis}}| \sim \frac{1}{\Lambda^{d-2}}\mathcal{O}$. IR SUSY breaking is suppressed. Should check whether leads to symmetry breaking or not!

• Power law and logarithmic behavior possible in both free and interacting IR theories. For phenomenology may be of interest to find free IR theory with power-law suppression (although in tension with need for Yukawas). This might lead to more robust IR stop mass suppression than in present models.

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• Another issue is that SQCD is sensitive to the UV soft masses, e.g.

$$\delta S_{UV}|_{SSB} = \int d^4x \ \left(\frac{m^2}{2}\left(Q\bar{Q} + \tilde{Q}\bar{\tilde{Q}}\right) + m'^2\left(Q\bar{Q} - \tilde{Q}\bar{\tilde{Q}}\right)\right) \ , \quad (19)$$

where m' multiplies the bottom component of the Baryon number current.

• In the IR, this leads to tachyons at leading order in m'

$$\delta S_{IR}|_{SSB} = \int d^4x \ m'^2 \left(\frac{N_c}{N_f - N_c}\right)\left(q\bar{q} - \tilde{q}\bar{\tilde{q}}\right) + \cdots \ . \quad (20)$$

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• Many authors have observed this behavior. Heuristically one expects it since the mass terms corresponding to conserved currents do not acquire anomalous dimension (at leading order in the soft deformation) and so we expect them to remain in the IR at leading order.

• While the statement about vanishing anomalous dimensions is true, the conclusion above about the IR behavior is not always correct!

• Heuristically, this is because the states that currents act on can be massive $\Rightarrow$ the currents decouple (and so do the corresponding soft terms at leading order).

Example: SUSY Breaking in SQCD and Light Stops (cont...)

• For example, consider SQCD with $N_f = 3N_c - 1$; turning on a mass for the first flavor

$$\delta W = mQ_1\tilde{Q}_1 \tag{21}$$

renders current, $\hat{J}_{11}$, acting (only) on these superfields w/ opposite phases massive—i.e., $\hat{J}_{11} \to 0$ and soft terms proportional to $\hat{J}_{11}| \to 0$ at leading order! Dynamical examples too.

• We want to study SUSY RG flows with small SUSY breaking of the form

$$\delta S_{UV}|_{SSB} = -\int d^4x \, \left(\lambda_U \cdot U_{\text{vis}}^{UV}| + \lambda_a \cdot \hat{J}_a^{UV}| + \lambda_A \cdot J_A^{UV}|\right) \, . \tag{22}$$

So, what's the general rule for RG behavior in our class of theories? In general additional relevant operators. Specialize to above. Currents $\Rightarrow$ powerful constraints.

The IR Behavior of Conserved Currents

• Want to understand how conserved current soft terms, $m\hat{J}_a|$, behave in IR. Therefore, study corresponding conserved currents.

• If want emergent SUSY and light sparticles from strong dynamics, want $U^{IR}_{\text{vis}} \to 0$. This amounts to studying theories with

$$\tilde{R}^{IR}_{\mu} = \lim_{E\to 0} \mathcal{R}_{\mu,\text{vis}} \ . \tag{23}$$

• In fact, for the theorem we will prove we don't even need an FZ multiplet and so can study

$$\tilde{R}^{IR}_{\mu} = \lim_{E\to 0} \mathcal{R}_{\mu}, \ \bar{D}^{\dot{\alpha}}\mathcal{R}_{\alpha\dot{\alpha}} = \chi_{\alpha}, \ \bar{D}_{\dot{\alpha}}\chi_{\alpha} = D^{\alpha}\chi_{\alpha} - \bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = 0 \ . \tag{24}$$

When discuss soft terms continue to assume existence of U.

The IR Behavior of Conserved Currents (cont...)

- Under these conditions can state a theorem for unitary QFTs:

- **Theorem:** A necessary and sufficient condition for the unbroken non-R currents, $\widehat{J}_a$, to flow to zero in the deep IR is that all the 't Hooft anomalies involving these currents vanish, i.e.

$$\mathrm{Tr}\widehat{J}_a\widehat{J}_b\widehat{J}_c = 0, \quad \mathrm{Tr}R\widehat{J}_a\widehat{J}_b = 0, \quad \mathrm{Tr}R^2\widehat{J}_a = 0, \quad \mathrm{Tr}\widehat{J}_a = 0 \ . \quad (25)$$

The IR Behavior of Conserved Currents (cont...)

• **Proof:** Necessity is trivial and follows from the following observation: if one of these anomalies is non-zero, 't Hooft anomaly matching forces there to be light fields charged under the corresponding symmetries. Sufficiency follows from the following reasoning. First, recall from (24) that R_μ flows to the IR superconformal R current, $\tilde{R}_\mu^{IR}$. Let us then suppose that $\hat{J}_a \to \hat{J}_a^{IR} \neq 0$. In this case, $\text{Tr}\tilde{R}_{IR}\hat{J}_a\hat{J}_a < 0$ (by unitarity) and so we must have $\text{Tr}R\hat{J}_a\hat{J}_a < 0$. This inequality conflicts with the second equation in (25), and so it must be the case that $\hat{J}_a \to 0$ in the IR. **q.e.d.**

The IR Behavior of Conserved Currents (cont...)

• $\mathcal{R} \to 0$ and $\hat{J} \to 0$ on different footing.

• **Corollary:** Consider the set of asymptotically free theories with simple gauge group and vanishing superpotential such that (24) holds. If such a theory has a set of non-anomalous global symmetries, $\hat{J}_a$, then it follows that a non-trivial subgroup of this symmetry will remain in the IR.

• **Sketch of proof:** We prove by contradiction. Using the theorem we should try to impose that the 't Hooft anomalies in (25) all vanish. A simple exercise in linear algebra reveals that this is inconsistent with the fact that our theory has a non-anomalous R symmetry.

Consequences for Soft Masses

• Leading order soft masses proportional to $m^2 \hat{J}_a|$ vanish if and only if all the corresponding 't Hooft anomalies vanish.

• Corollary assures us that we get leading order soft masses in generic asymptotically free theories unless we restrict the set of UV soft terms to a subset of measure zero since (at least for simple gauge group and vanishing superpotential)

$$\mathcal{G} = U(1)_R \times U(1)^{s-1} \times \prod_{i=1}^{s} SU(n_i) \ . \tag{26}$$

$\Rightarrow$ have to often make assumptions about SUSY breaking and mediation mechanism. Large deformations of these theories often lead to incalculable soft terms [Abel, M.B., Komargodski, '11].

Consequences for Soft Masses

• **Upshot:** If we want natural and calculable theories of emergent SUSY and light sparticles, it is perhaps more promising to start from an interacting UV fixed point since can have less symmetry.

• Interacting fixed point should have some global symmetry group $\mathcal{G} \supset \mathcal{G}_{SSM}$ (that is weakly gauged) if we want light emergent SSM states.

• Want a minimal embedding so as to avoid too much additional global symmetry; generically will need relevant deformations to then get rid of exotics; these deformations generically break global symmetries and lead to incalculable soft terms.

Consequences for Soft Masses (cont...)

• To have a space of couplings one can play with that don't break global symmetries it is natural to consider conformal manifold (i.e, the space of exactly marginal deformations), $\mathcal{M}$ (these are often present in interacting $\mathcal{N}=1$ theories)

$$\mathcal{M} = \{\lambda^i\}/\mathcal{G}^{\mathbf{C}} \ , \tag{27}$$

where the λ^i are the marginal (holomorphic) couplings [`Green et. al., '10`]. Want submanifold of $\mathcal{G}^{\mathbf{C}}$ singlets, $\widehat{\mathcal{M}}$.

• What is the phenomenological role of the conformal manifold?

• Nice global symmetry group for a conformal manifold that might allow EWSB and light Higgs: $\mathcal{G} = SU(2) \times SU(2) \times U(1)$. String theorists will recognize this as the global symmetry group of the conifold...

Consequences for Soft Masses (cont...)

• We have seen how general principles of the RG flow (unitarity and 't Hooft anomaly matching) as well as a conjectured property of a large class of four-dimensional R-symmetric RG flows may provide constraints on theories of light sparticles.

• Clearly a lot remains to be explored, but careful analysis of a few well-defined QFT degrees of freedom may yield interesting constraints that (hopefully) help us with our phenomenological searches at the LHC.