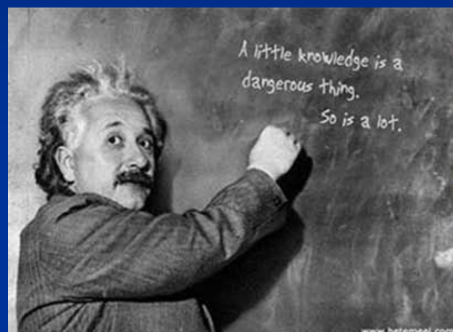
Quantum Gravity and emergent metric

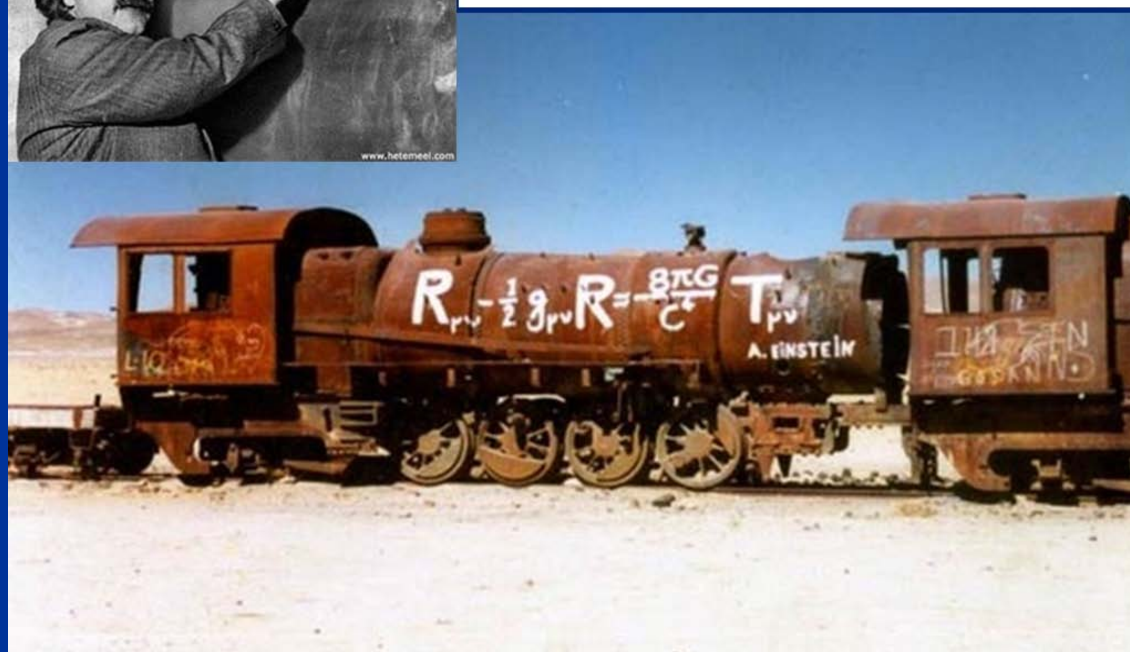
Why do we need quantum gravity for cosmology ?

- gravitational equations provide fundamental framework for cosmology
- gravity is coupled to quantum matter and radiation
- energy momentum tensor is a quantum object
- can one have an equation with classical metric field on one side and a quantum object on the other side ?

Can one have an equation with classical metric field on one side and a quantum object on the other side ?



$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$



yes : equation for expectation values !

metric is expectation value of quantum field

- One only needs to assume that **some** quantum theory exists for which an observable with properties of metric exists and has a nonzero expectation value
- formalism : quantum effective action – exact field equations follow from variation of action functional
- **If** effective action takes form of Einstein-Hilbert action (with cosmological constant) the Einstein field equations follow
- This would be sufficient for cosmology !

Einstein gravity

- Is Einstein Hilbert action sufficient ?
- It cannot be the exact effective action for a theory of quantum gravity !
- Can it be a sufficiently accurate approximation for the quantum effective action ?
- Answer to this question needs consistent theory of **quantum gravity** !

Einstein gravity as effective theory for large distance scales or small momenta

- diffeomorphism symmetry
- derivative expansion

  zero derivatives : cosmological constant

  two derivatives : curvature scalar R

  four derivatives : R^2, two more tensor structures

higher derivatives are expected to be induced by quantum fluctuations

short distance modifications

coefficient R^2 order one

(typical quantum contribution $1/16\pi^2$) :

higher order derivative terms play a role only once curvature scalar is of the order of squared Planck mass

singularity of black holes , inflationary cosmology

no analytic behavior expected : $R^2 \ln(R)$ etc.

long distance modifications ??

- non- local terms
- f(R) with huge coefficients of Taylor expansion

this could modify late time behavior of cosmology and be related to dark energy

possible explanation why cosmological constant is zero or small ?

need for quantum gravity

- before judgment one needs at least one consistent model of quantum gravity
- will it be unique ? probably not !

Quantum gravity

- Quantum field theory
- Functional integral formulation

Symmetries are crucial

- Diffeomorphism symmetry

  (invariance under general coordinate transformations)
- Gravity with fermions : local Lorentz symmetry

Degrees of freedom less important :

metric, vierbein , spinors , random triangles , conformal fields…

Graviton , metric : collective degrees of freedom in theory with diffeomorphism symmetry

Regularized quantum gravity

① For finite number of lattice points : functional integral should be well defined

② Lattice action invariant under local Lorentz-transformations

③ Continuum limit exists where gravitational interactions remain present

④ Diffeomorphism invariance of continuum limit , and geometrical lattice origin for this

scalar gravity

with D.Sexty

- quantum field theory for scalars
- d=2 , two complex fields i=1,2
- non-linear sigma-model $\sum_i \varphi_i^* \varphi_i = 1$

- diffeomorphism symmetry of action

$$S = \beta \int d^2 x \epsilon^{\mu\nu} (\varphi_1^* \partial_\mu \varphi_1 - \varphi_1 \partial_\mu \varphi_1^*)(\varphi_2^* \partial_\nu \varphi_2 - \varphi_2 \partial_\nu \varphi_2^*)$$

lattice regularization

$$S = \beta \int d^2x \epsilon^{\mu\nu} (\varphi_1^* \partial_\mu \varphi_1 - \varphi_1 \partial_\mu \varphi_1^*)(\varphi_2^* \partial_\nu \varphi_2 - \varphi_2 \partial_\nu \varphi_2^*)$$

$$\partial_\mu \varphi(\tilde{y}) = \frac{1}{2\Delta} [\varphi(\tilde{y} + v_\mu) - \varphi(\tilde{y} - v_\mu)]$$

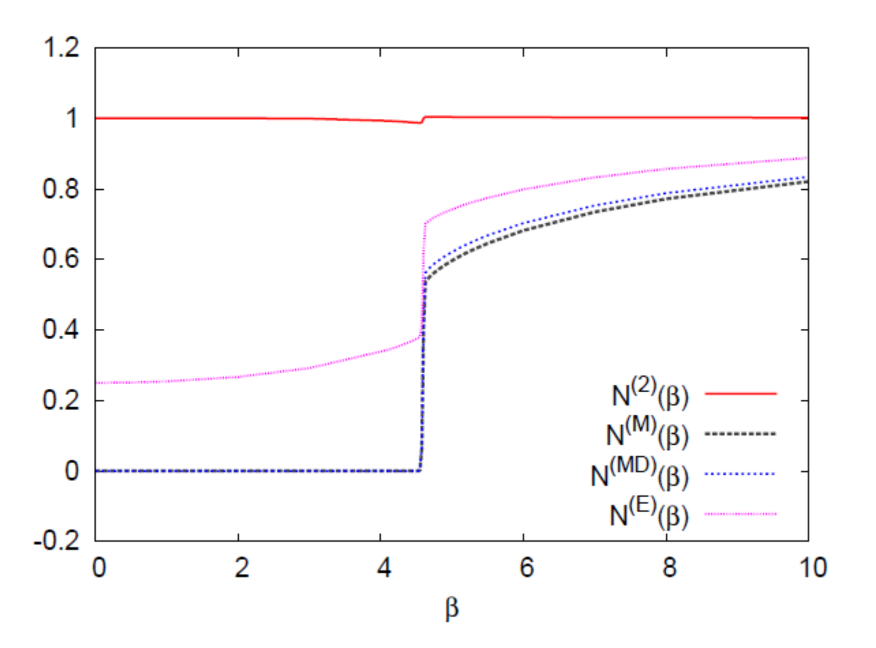
$$(\tilde{v}_\nu)^\mu = \delta_\nu^\mu$$

$$\bar{\varphi}(\tilde{y}) = \frac{1}{4} [\varphi(\tilde{y} + v_0) + \varphi(\tilde{y} - v_0) + \varphi(\tilde{y} + v_1) + \varphi(\tilde{y} - v_1)]$$

collective metric observable

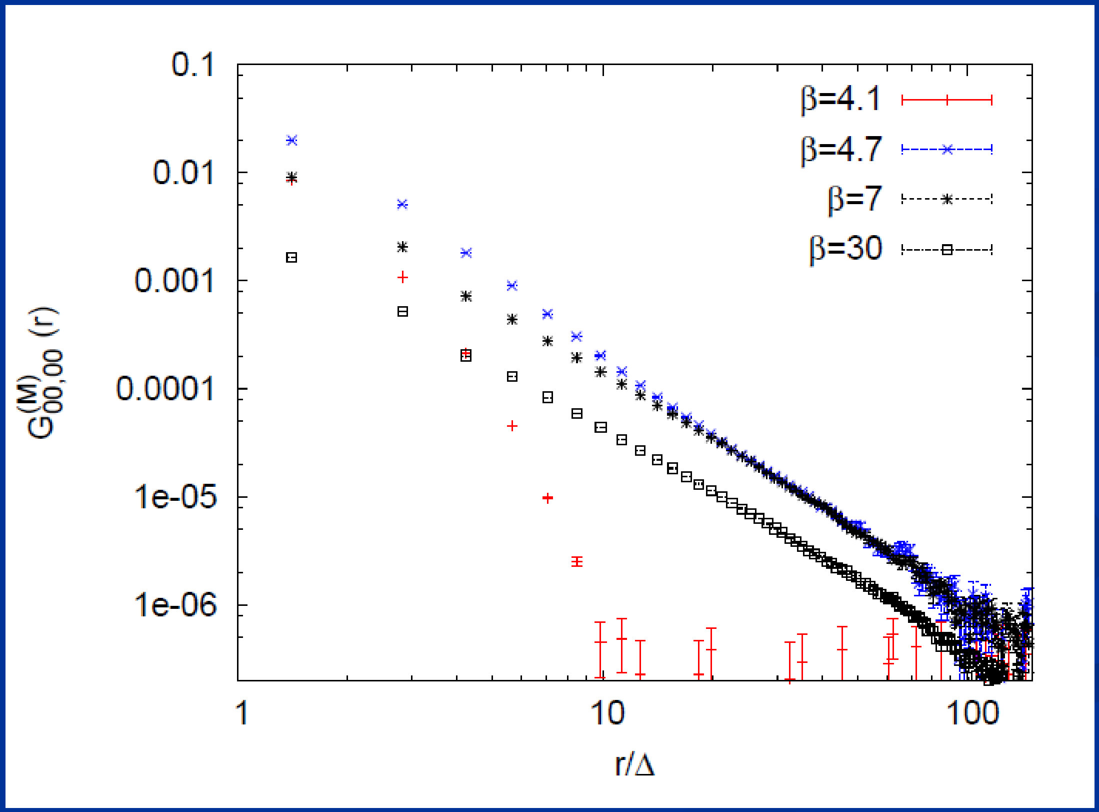
$$\tilde{g}^{(M)}_{\mu\nu} = 8\Delta^2 Re(i\varphi_1^*\partial_\mu\varphi_1)Re(i\varphi_2^*\partial_\nu\varphi_2) + (\varphi_1 \leftrightarrow \varphi_2)$$

$$g^{(M)}_{\mu\nu} = \langle \tilde{g}^{(M)}_{\mu\nu} \rangle = N^{(M)}(\beta)\eta_{\mu\nu}$$



metric correlation function

$$G_{\mu\nu,\rho\sigma}(x,y) = \langle h_{\mu\nu}(x) h_{\rho\sigma}(y) \rangle$$



response of metric to source

$$h_{\mu\nu}(x) = \int_y G_{\mu\nu,\rho\sigma}(x,y)t^{\rho\sigma}(y)$$

$$t^{00}(y^0, y^1) = M\delta(y^1)$$

$$h_{00}(x_0, x_1) \sim \frac{M}{|x_1|}$$

Spinor gravity

is formulated in terms of fermions

Unified Theory
of fermions and bosons

Fermions fundamental
Bosons collective degrees of freedom

- Alternative to supersymmetry
- Graviton, photon, gluons, W-,Z-bosons , Higgs scalar : all are collective degrees of freedom (composite)
- Composite bosons look fundamental at large distances, e.g. hydrogen atom, helium nucleus, pions
- Characteristic scale for compositeness : Planck mass

Massless collective fields or bound states – familiar if dictated by symmetries

for chiral QCD :

Pions are massless bound states of massless quarks !

for strongly interacting electrons :

antiferromagnetic spin waves

Geometrical degrees of freedom

- Ψ(x) : spinor field (Grassmann variable)
- vielbein : fermion bilinear

$$\tilde{E}_\mu^m = i\bar{\psi}\gamma^m\partial_\mu\psi$$

$$E_\mu^m(x) = \langle \tilde{E}_\mu^m(x) \rangle$$

Emergence of geometry

vierbein $\langle \tilde{E}_\mu^{(M)m} \rangle = \langle (\tilde{E}_\mu^{M)m})^* \rangle = e_\mu^m$ / Δ

metric $g_{\mu\nu} = e_\mu^m e_\nu^n \eta_{mn}$

Possible Action

$$S_E \sim \int d^d x \, \det\left(\tilde{E}_\mu^m(x)\right)$$

$$\tilde{E} = \frac{1}{d!} \epsilon^{\mu_1 \dots \mu_d} \epsilon_{m_1 \dots m_d} \tilde{E}_{\mu_1}^{m_1} \dots \tilde{E}_{\mu_d}^{m_d} = \det(\tilde{E}_\mu^m)$$

contains 2d powers of spinors

d derivatives contracted with ε - tensor

$$\tilde{E}_\mu^m = i\bar{\psi}\gamma^m \partial_\mu \psi$$

Symmetries

- General coordinate transformations (diffeomorphisms)
- Spinor $\psi(x)$: transforms as scalar
- Vielbein $\tilde{E}_\mu^m = i\bar{\psi}\gamma^m\partial_\mu\psi$: transforms as vector
- Action S : invariant

K.Akama, Y.Chikashige, T.Matsuki, H.Terazawa (1978)
K.Akama (1978)
D.Amati, G.Veneziano (1981)
G.Denardo, E.Spallucci (1987)
A.Hebecker, C.Wetterich

Lorentz- transformations

Global Lorentz transformations:

- spinor ψ
- vielbein transforms as vector
- action invariant

Local Lorentz transformations:

- vielbein does **not** transform as vector
- inhomogeneous piece, missing covariant derivative

$$\tilde{E}_\mu^m = i\bar{\psi}\gamma^m\partial_\mu\psi$$

Two alternatives :

1) Gravity with **global** and not local Lorentz symmetry ?
Compatible with observation !

2) Action with
local Lorentz symmetry ?
Can be constructed !

Spinor gravity with local Lorentz symmetry

Spinor degrees of freedom

- Grassmann variables ψ_γ^a
- Spinor index $\gamma = 1 \ldots 8$
- Two flavors $a = 1, 2$
- Variables at every space-time point

$$x^\mu = (x^0, x^1, x^2, x^3)$$

- Complex Grassmann variables

$$\varphi_\alpha^a(x) = \psi_\alpha^a(x) + i\psi_{\alpha+4}^a(x)$$

Action with local Lorentz symmetry

$$S = \alpha \int d^4x A^{(8)} D + c.c.$$

A : product of
all eight spinors ,
maximal number ,
totally antisymmetric

$$A^{(8)} = \frac{1}{8!}\epsilon_{\epsilon_1\epsilon_2\dots\epsilon_8}\varphi_{\epsilon_1}\dots\varphi_{\epsilon_8}$$
$$= \frac{1}{(24)^2}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\varphi^1_{\alpha_1}\dots\varphi^1_{\alpha_4}\epsilon_{\beta_1\beta_2\beta_3\beta_4}\varphi^2_{\beta_1}\dots\varphi^2_{\beta_4}$$
$$= \varphi^1_1\varphi^1_2\varphi^1_3\varphi^1_4\varphi^2_1\varphi^2_2\varphi^2_3\varphi^2_4$$

D : antisymmetric product
of four derivatives ,
L is totally symmetric
Lorentz invariant tensor

$$D = \epsilon^{\mu_1\mu_2\mu_3\mu_4}\partial_{\mu_1}\varphi_{\eta_1}\partial_{\mu_2}\varphi_{\eta_2}\partial_{\mu_3}\varphi_{\eta_3}\partial_{\mu_4}\varphi_{\eta_4}L_{\eta_1\eta_2\eta_3\eta_4}$$

Double index $\eta = (\beta, b)$

Symmetric four-index invariant

Symmetric invariant bilinears

$$S^{\pm}_{\eta_1\eta_2} = (S^{\pm})^{b_1b_2}_{\beta_1\beta_2} = \mp(C_{\pm})_{\beta_1\beta_2}(\tau_2)^{b_1b_2}$$

Lorentz invariant tensors

$$C_+ = \frac{1}{2}(C_1 + C_2) = \frac{1}{2}C_1(1+\bar{\gamma}) = \begin{pmatrix} \tau_2 & ; & 0 \\ 0 & ; & 0 \end{pmatrix},$$
$$C_- = \frac{1}{2}(C_1 - C_2) = \frac{1}{2}C_1(1-\bar{\gamma}) = \begin{pmatrix} 0 & ; & 0 \\ 0 & ; & -\tau_2 \end{pmatrix}$$

Symmetric four-index invariant

$$L_{\eta_1\eta_2\eta_3\eta_4} = \frac{1}{6}(S^+_{\eta_1\eta_2}S^-_{\eta_3\eta_4} + S^+_{\eta_1\eta_3}S^-_{\eta_2\eta_4} + S^+_{\eta_1\eta_4}S^-_{\eta_2\eta_3}$$
$$+S^+_{\eta_3\eta_4}S^-_{\eta_1\eta_2} + S^+_{\eta_2\eta_4}S^-_{\eta_1\eta_3} + S^+_{\eta_2\eta_3}S^-_{\eta_1\eta_4})$$

Two flavors needed in four dimensions for this construction

Weyl spinors

$$\varphi_+ = \frac{1}{2}(1+\bar{\gamma})\varphi \ , \ \varphi_- = \frac{1}{2}(1-\bar{\gamma})\varphi$$

$$\bar{\gamma} = -\gamma^0\gamma^1\gamma^2\gamma^3$$ = diag (1 , 1 , -1 , -1)

$$\gamma^0 = \tau_1 \otimes 1 \ , \ \gamma^k = \tau_2 \otimes \tau_k .$$

Action in terms of Weyl - spinors

$$S = \alpha \int d^4x \epsilon^{\mu_1\mu_2\mu_3\mu_4} F^+_{\mu_1\mu_2} F^-_{\mu_3\mu_4} + c.c.$$

$$F^{\pm}_{\mu_1\mu_2} = A^{\pm} D^{\pm}_{\mu_1\mu_2}$$

$$A^+ = \varphi^1_{+1}\varphi^1_{+2}\varphi^2_{+1}\varphi^2_{+2}$$

$$D^{\pm}_{\mu_1\mu_2} = \partial_{\mu_1}\varphi_{\eta_1} S^{\pm}_{\eta_1\eta_2} \partial_{\mu_2}\varphi_{\eta_2}$$

Relation to previous formulation

$$A^{(8)} = A^+ A^-$$

$$D = \epsilon^{\mu_1\mu_2\mu_3\mu_4} D^+_{\mu_1\mu_2} D^-_{\mu_3\mu_4}$$

$$S = \alpha \int d^4x A^{(8)} D + c.c.$$

SO(4,C) - symmetry

$$\delta\varphi^a_\alpha(x) = -\frac{1}{2}\epsilon_{mn}(x)(\Sigma^{mn}_E)_{\alpha\beta}\varphi^a_\beta(x)$$

$$\Sigma^{mn}_E = -\frac{1}{4}[\gamma^m_E, \gamma^n_E] \ , \ \{\gamma^m_E, \gamma^n_E\} = 2\delta^{mn}$$

Action invariant for arbitrary
complex *transformation parameters ε !*

Real ε : SO (4) - transformations

Signature of time

Difference in signature between
space and time :

only from spontaneous symmetry breaking ,
e.g. by
expectation value of vierbein – bilinear !

Minkowski - action

$$S = -iS_M \; , \; e^{-S} = e^{iS_M}$$

Action describes **simultaneously** euclidean and Minkowski theory !

SO (1,3) transformations : $\epsilon_{0k} = -i\epsilon_{0k}^{(M)}$ $\epsilon_{kl}^{(M)} = \epsilon_{kl}$

$$\delta\varphi = -\frac{1}{2}\epsilon_{mn}^{(M)}\Sigma_M^{mn}\varphi,$$

$$\Sigma_M^{mn} = -\frac{1}{4}[\gamma_M^m, \gamma_M^n] \; , \; \{\gamma_M^m, \gamma_M^n\} = \eta^{mn}$$

$$\gamma_M^0 = -i\gamma_E^0, \gamma_M^k = \gamma_E^k$$

Emergence of geometry

Euclidean vierbein bilinear

$$\tilde{E}_\mu^m = \varphi^a C \gamma^m \partial_\mu \varphi^b V^{ab} = -\partial_\mu \varphi^a C \gamma^m \varphi^b V^{ab}$$

Minkowski -
vierbein bilinear

$$\tilde{E}_\mu^{(M)m} = \varphi V C \gamma_M^m \partial_\mu \varphi$$

$$\tilde{E}_\mu^{(M)0} = -i \tilde{E}_\mu^0 \ , \ \tilde{E}_\mu^{(M)k} = \tilde{E}_\mu^k.$$

Global
Lorentz - transformation

$$\delta \tilde{E}_\mu^{(M)m} = -\tilde{E}_\mu^{(M)n} \epsilon_n^{(M)m}$$

vierbein $\langle \tilde{E}_\mu^{(M)m} \rangle = \langle (\tilde{E}_\mu^{M)m})^* \rangle = \quad e_\mu^m \ / \ \Delta$

metric $g_{\mu\nu} = e_\mu^m e_\nu^n \eta_{mn}$

Can action can be reformulated in terms of vierbein bilinear ?

$$S = \alpha \int d^4x W \det(\tilde{E}_\mu^m) + c.c.,$$

No suitable W exists

How to get gravitational field equations ?

How to determine geometry of space-time, vierbein and metric ?

Functional integral formulation of gravity

- Calculability
  (at least in principle)
- Quantum gravity
- Non-perturbative formulation

$$
\begin{aligned}
Z &= \int \mathcal{D}\psi g_f \exp(-S) g_{in}, \\
\int \mathcal{D}\psi &= \prod_x \prod_{a=1}^{2} \{ \int d\psi_1^a(x) \dots \int d\psi_8^a(x) \}
\end{aligned}
$$

$$
\langle \mathcal{A} \rangle = Z^{-1} \int \mathcal{D}\psi g_f \mathcal{A} \exp(-S) g_{in}.
$$

Vierbein and metric

$$E^m_\mu(x) = \langle \tilde{E}^m_\mu(x) \rangle$$

$$g_{\mu\nu}(x) = E^m_\mu(x) E_{\nu m}(x)$$

Generating functional

$$Z[J] = \int \mathcal{D}\psi \exp\left\{ -(S + S_J) \right\}$$

$$S_J = -\int d^d x J^\mu_m \tilde{E}^m_\mu$$

$$E^m_\mu(x) = \langle \tilde{E}^m_\mu(x) \rangle = \frac{\delta \ln Z}{\delta J^\mu_m(x)}$$

If regularized functional measure can be defined (consistent with diffeomorphisms)

Non- perturbative definition of quantum gravity

$$Z[J] = \int \underline{\mathcal{D}\psi} \exp\left\{-(S + S_J)\right\}$$

Effective action

$$\Gamma[E^m_\mu] = -W[J^\mu_m] + \int d^dx\, J^\mu_m E^m_\mu$$

W=ln Z

Gravitational field equation for vierbein

$$\frac{\delta\Gamma}{\delta E^m_\mu} = J^\mu_m$$

similar for metric

Gravitational field equation and energy momentum tensor

$$\frac{\delta \Gamma}{\delta E^m_\mu} = J^\mu_m$$

$$T^{\mu\nu} = E^{-1} E^{m\mu} J^\nu_m$$

Special case : effective action depends only on metric

$$\Gamma'_0[E^m_\mu] = \Gamma'_0\left[g_{\nu\rho}[E^m_\mu]\right]$$

$$g_{\mu\nu} = E^m_\mu E_{\nu m}$$

$$T^{\mu\nu}_{(g)} = -\frac{2}{\sqrt{g}} \frac{\delta \Gamma'_0}{\delta g_{\mu\nu}}$$

$$T^{\mu\nu} = -E^{-1} E^{m\mu} \frac{\delta \Gamma'_0}{\delta g_{\rho\sigma}} \frac{\delta g_{\rho\sigma}}{\delta E^m_\nu} = T^{\mu\nu}_{(g)}$$

Symmetries dictate general form of effective action and gravitational field equation

diffeomorphisms !

Effective action for metric :
curvature scalar R + additional terms

Lattice spinor gravity

Lattice regularization

- Hypercubic lattice
- Even sublattice $y^\mu = \tilde{y}^\mu \Delta, \ \tilde{y}^\mu \text{ integer}, \ \Sigma_\mu \tilde{y}^\mu \text{ even}$
- Odd sublattice $z^\mu = \tilde{z}^\mu \Delta \ , \ \tilde{z}^\mu \text{ integer}, \ \Sigma_\mu \tilde{z}^\mu \text{ odd}$

- Spinor degrees of freedom on points of odd sublattice

Lattice action

- Associate cell to each point y of even sublattice
- Action: sum over cells

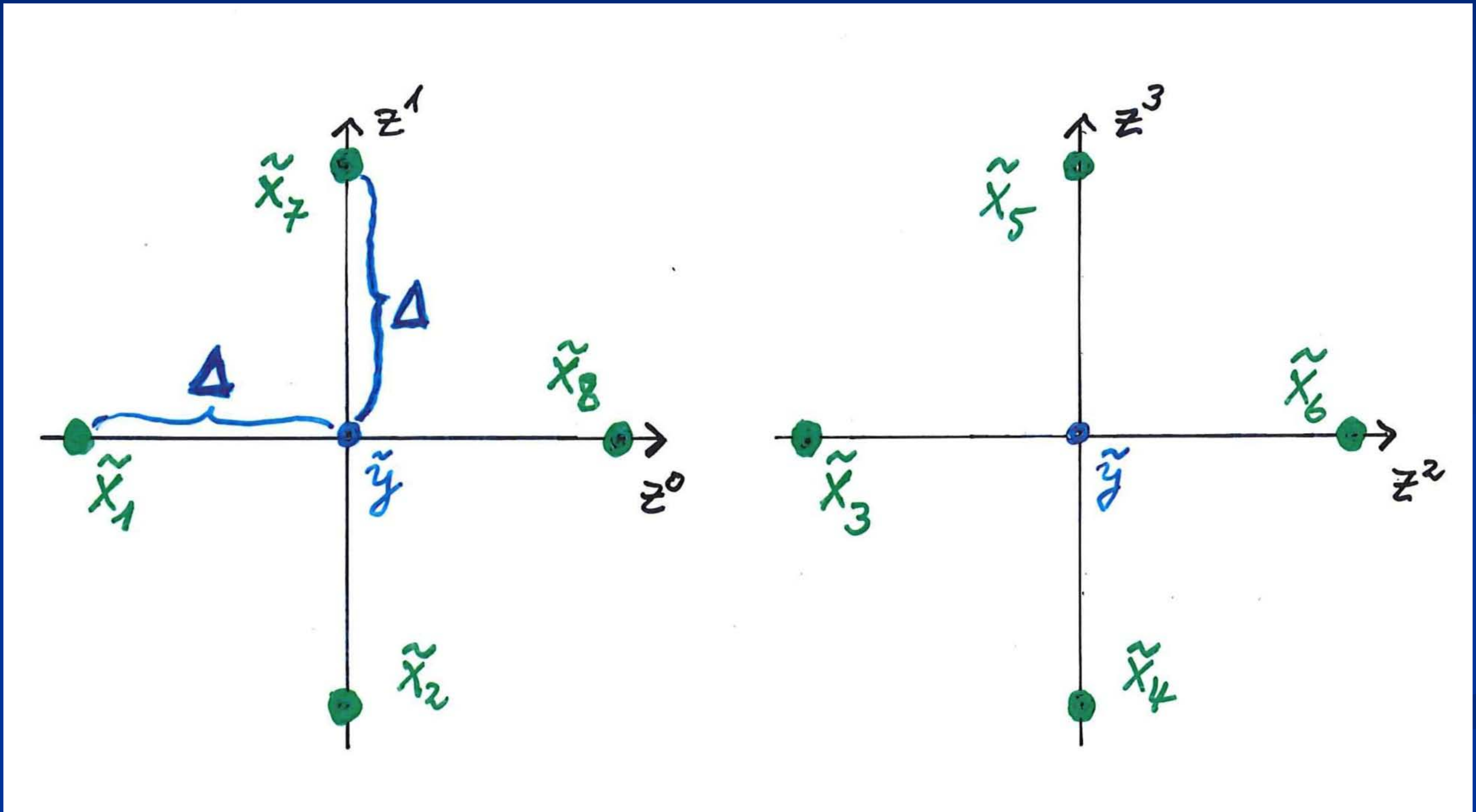
$$S = \tilde{\alpha} \sum_y \mathcal{L}(y) + c.c.$$

- For each cell : twelve spinors located at nearest neighbors of y (on odd sublattice)

$$\tilde{z}^\mu \left(\tilde{x}_j(\tilde{y})\right) = \tilde{y}^\mu + V_j^\mu$$

$$\begin{array}{ll} V_1 = (-1,0,0,0) \;, & V_5 = (0,0,0,1) \\ V_2 = (0,-1,0,0) \;, & V_6 = (0,0,1,0) \\ V_3 = (0,0,-1,0) \;, & V_7 = (0,1,0,0) \\ V_4 = (0,0,0,-1) \;, & V_8 = (1,0,0,0) \end{array}$$

cells



z^1
$\tilde{x}_7$
Δ
Δ
$\tilde{x}_8$
$\tilde{x}_1$
$\tilde{y}$
z^0
$\tilde{x}_2$
z^3
$\tilde{x}_5$
$\tilde{x}_6$
$\tilde{x}_3$
$\tilde{y}$
z^2
$\tilde{x}_4$

Local SO (4,C) symmetry

Basic SO(4,C) invariant building blocks

$$\tilde{\mathcal{H}}^k_\pm(\tilde{x}) = \varphi^a_\alpha(\tilde{x})(C_\pm)_{\alpha\beta}(\tau_2\tau_k)^{ab}\varphi^b_\beta(\tilde{x})$$

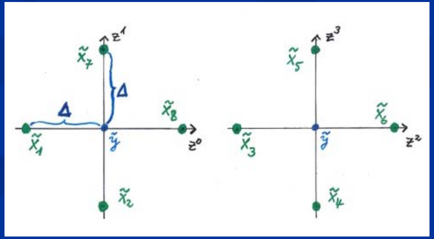
Lattice action

$$\begin{aligned}\mathcal{L}(y) \; &= \; \frac{1}{6}\{\mathcal{F}_+^{1,2,8,7}\mathcal{F}_-^{3,4,6,5} + \mathcal{F}_+^{1,3,8,6}\mathcal{F}_-^{7,4,2,5} \\ &\quad +\mathcal{F}_+^{1,4,8,5}\mathcal{F}_-^{3,7,6,2} + (\mathcal{F}_+ \leftrightarrow \mathcal{F}_-)\}.\end{aligned}$$

$$\mathcal{F}_\pm^{abcd} = \frac{1}{24}\epsilon^{klm}\,[\tilde{\mathcal{H}}^k_\pm(\tilde{x}_a)\tilde{\mathcal{H}}^l_\pm(\tilde{x}_b)\tilde{\mathcal{H}}^m_\pm(\tilde{x}_c)$$

$$+\tilde{\mathcal{H}}^k_\pm(\tilde{x}_b)\tilde{\mathcal{H}}^l_\pm(\tilde{x}_c)\tilde{\mathcal{H}}^m_\pm(\tilde{x}_d) + \tilde{\mathcal{H}}^k_\pm(\tilde{x}_c)\tilde{\mathcal{H}}^l_\pm(\tilde{x}_d)\tilde{\mathcal{H}}^m_\pm(\tilde{x}_a)$$

$$+\tilde{\mathcal{H}}^k_\pm(\tilde{x}_d)\tilde{\mathcal{H}}^l_\pm(\tilde{x}_a)\tilde{\mathcal{H}}^m_\pm(\tilde{x}_b)]$$



Lattice symmetries

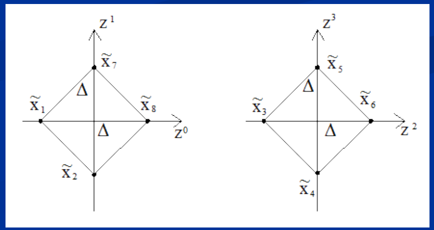
- Rotations by π/2 in all lattice planes (invariant)

$$\mathcal{F}_{\pm}^{abcd} = \mathcal{F}_{\pm}^{bcda} = \mathcal{F}_{\pm}^{cdab} = \mathcal{F}_{\pm}^{dabc}.$$

- Reflections of all lattice coordinates (odd)

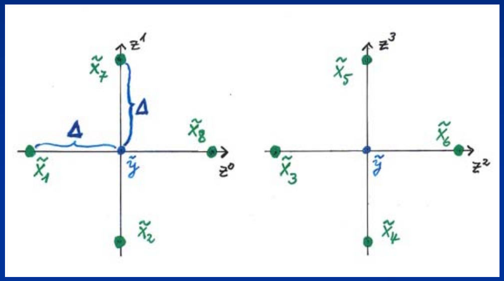
$$\mathcal{F}_{\pm}^{cbad} = \mathcal{F}_{\pm}^{adcb} = -\mathcal{F}_{\pm}^{abcd}$$

- Diagonal reflections e.g $z_1 \leftrightarrow z_2$ (odd)



Lattice derivatives

$$
\begin{aligned}
\hat{\partial}_0\varphi(y) &= \frac{1}{2\Delta}\left(\varphi(\tilde{x}_8)-\varphi(\tilde{x}_1)\right)\\
\hat{\partial}_1\varphi(y) &= \frac{1}{2\Delta}\left(\varphi(\tilde{x}_7)-\varphi(\tilde{x}_2)\right)\\
\hat{\partial}_2\varphi(y) &= \frac{1}{2\Delta}\left(\varphi(\tilde{x}_6)-\varphi(\tilde{x}_3)\right)\\
\hat{\partial}_3\varphi(y) &= \frac{1}{2\Delta}\left(\varphi(\tilde{x}_5)-\varphi(\tilde{x}_4)\right)
\end{aligned}
$$



and cell averages

$$
\begin{aligned}
\bar{\varphi}_0(y) &= \frac{1}{2}\left(\varphi(\tilde{x}_1)+\varphi(\tilde{x}_8)\right)\ ,\ \ \bar{\varphi}_1(y) = \frac{1}{2}\left(\varphi(\tilde{x}_2)+\varphi(\tilde{x}_7)\right)\\
\bar{\varphi}_2(y) &= \frac{1}{2}\left(\varphi(\tilde{x}_3)+\varphi(\tilde{x}_6)\right)\ ,\ \ \bar{\varphi}_3(y) = \frac{1}{2}\left(\varphi(\tilde{x}_4)+\varphi(\tilde{x}_5)\right)
\end{aligned}
$$

express spinors in terms of derivatives and averages

$$\varphi(\tilde{x}_j) = \sigma_j^{\mu}\bar{\varphi}_{\mu} + V_j^{\mu}\Delta\hat{\partial}_{\mu}\varphi$$

$$\sigma_j^{\mu} = (V_j^{\mu})^2$$

Bilinears and lattice derivatives

$$\mathcal{H}^k_{\pm}(\tilde{x}_j) = \sigma^\mu_j \bar{\mathcal{H}}^k_{\pm\mu}(y) + 2\Delta V^\mu_j \tilde{\mathcal{D}}^k_{\pm\mu}(y) + \Delta^2 \sigma^\mu_j \mathcal{G}^k_{\pm\mu}(y).$$

$$\tilde{\mathcal{D}}^k_{\pm\mu} = (\bar{\varphi}_\mu)^a_\alpha (C_\pm)_{\alpha\beta} (\tau_2\tau_k)^{ab} \hat{\partial}_\mu \varphi^b_\beta$$

$$\tilde{\mathcal{G}}^k_{\pm\mu} = \hat{\partial}_\mu \varphi^a_\alpha (C_\pm)_{\alpha\beta} (\tau_2\tau_k)^{ab} \hat{\partial}_\mu \varphi^b_\beta$$

$$\hat{\mathcal{H}}^k_{\pm\mu} = \bar{\mathcal{H}}^k_{\pm\mu} + \Delta^2 \tilde{\mathcal{G}}^k_{\pm\mu} \ , \ \mathcal{H}^k_{\pm ab} = \frac{1}{2}(\hat{\mathcal{H}}^k_{\pm a} + \hat{\mathcal{H}}^k_{\pm b}).$$

Action in terms of lattice derivatives

$$\mathcal{F}_{+}^{1,2,8,7} = \frac{2\Delta^2}{3}\epsilon^{klm}\mathcal{H}^k_{+01}(\tilde{\mathcal{D}}^l_{+0}\tilde{\mathcal{D}}^m_{+1} - \tilde{\mathcal{D}}^l_{+1}\tilde{\mathcal{D}}^m_{+0}).$$

$$\mathcal{F}^{\pm}_{01} = -\mathcal{F}^{\pm}_{10} = \mathcal{F}^{1,2,8,7}_{\pm}$$

$$\mathcal{F}^{\pm}_{\mu\nu} = \frac{2\Delta^2}{3}\epsilon^{klm}\mathcal{H}^k_{\pm\mu\nu}(\tilde{\mathcal{D}}^l_{\pm\mu}\tilde{\mathcal{D}}^m_{\pm\nu} - \tilde{\mathcal{D}}^l_{\pm\nu}\tilde{\mathcal{D}}^m_{\pm\mu})$$

$$\mathcal{L}(y) = \frac{1}{24}\epsilon^{\mu_1\mu_2\mu_3\mu_4}\mathcal{F}^{+}_{\mu_1\mu_2}\mathcal{F}^{-}_{\mu_3\mu_4}$$

$$\tilde{\mathcal{D}}^k_{\pm\mu} = (\bar{\varphi}_\mu)^a_\alpha(C_\pm)_{\alpha\beta}(\tau_2\tau_k)^{ab}\hat{\partial}_\mu\varphi^b_\beta$$

Continuum limit

$$\mathcal{L}(y) \to \frac{32}{3}\Delta^4 \epsilon^{\mu_1\mu_2\mu_3\mu_4} F^+_{\mu_1\mu_2} F^-_{\mu_3\mu_4}$$

$$\Delta^4 \Sigma_y = \frac{1}{2}\int_y$$

Lattice distance Δ drops out in continuum limit !

$$S = \frac{16}{3}\tilde{\alpha}\int_y \epsilon^{\mu_1\mu_2\mu_3\mu_4} F^+_{\mu_1\mu_2} F^-_{\mu_3\mu_4} + c.c$$

$$\tilde{\alpha} = 3\alpha/16$$

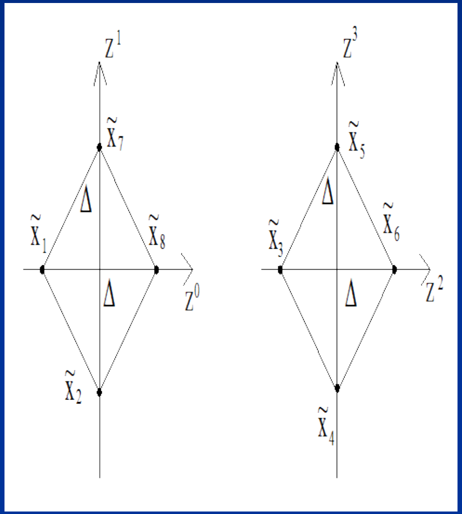
Regularized quantum gravity

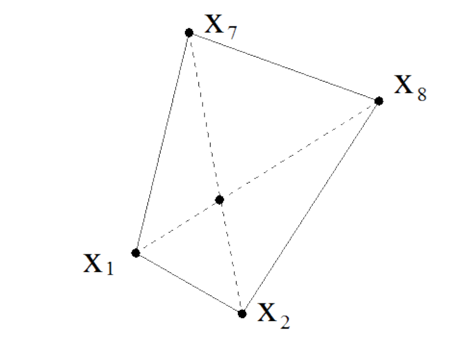
- For finite number of lattice points : functional integral should be well defined
- Lattice action invariant under local Lorentz-transformations
- Continuum limit exists where gravitational interactions remain present
- Diffeomorphism invariance of continuum limit , and geometrical lattice origin for this

Lattice diffeomorphism invariance

- Lattice equivalent of diffeomorphism symmetry in continuum
- Action does not depend on positioning of lattice points in manifold , once formulated in terms of lattice derivatives and average fields in cells
- Arbitrary instead of regular lattices
- Continuum limit of lattice diffeomorphism invariant action is invariant under general coordinate transformations

Positioning of lattice points





$$V(\tilde{y}) = \frac{1}{2}\epsilon_{\mu\nu}(x_4^\mu - x_1^\mu)(x_3^\nu - x_2^\nu)$$

$$\int d^2x = \sum_{\tilde{y}} V(\tilde{y})$$

Lattice action and functional measure of spinor gravity are lattice diffeomorphism invariant !

Next tasks

- Compute effective action for composite metric
- Verify presence of Einstein-Hilbert term

  (curvature scalar)

Conclusions

- Unified theory based only on fermions seems possible
- Quantum gravity –
  functional measure can be regulated
- Does realistic higher dimensional unified
  model exist ?

end

Lattice derivatives

$$\hat{\partial}_0 H_k(\tilde{y}) = \frac{1}{2V(\tilde{y})}\Big\{(x_3^1 - x_2^1)\big(H_k(\tilde{x}_4) - H_k(\tilde{x}_1)\big) - (x_4^1 - x_1^1)\big(H_k(\tilde{x}_3) - H_k(\tilde{x}_2)\big)\Big\},$$

$$\hat{\partial}_1 H_k(\tilde{y}) = \frac{1}{2V(\tilde{y})}\Big\{(x_4^0 - x_1^0)\big(H_k(\tilde{x}_3) - H_k(\tilde{x}_2)\big) - (x_3^0 - x_2^0)\big(H_k(\tilde{x}_4) - H_k(\tilde{x}_1)\big)\Big\}.$$

$$H_k(\tilde{x}_{j_1}) - H_k(\tilde{x}_{j_2}) = (x_{j_1}^{\mu} - x_{j_2}^{\mu})\hat{\partial}_{\mu} H_k(\tilde{y})$$

Cell average :

$$H_k(\tilde{y}) = \frac{1}{4}\sum_j H_k\big(\tilde{x}_j(\tilde{y})\big)$$

Lattice diffeomorphism invariance

$$S(x_p) = \int d^2x \bar{L}(\tilde{y}; x_p) = \int d^2x \bar{L}(x; x_p)$$

$$\bar{L}(\tilde{y}; x_p) = \bar{L}(x; x_p) = \frac{\hat{L}(\tilde{y}; x_p)}{V(\tilde{y}; x_p)}.$$

$$x'_p = x_p + \xi_p$$

$$\bar{L}(\tilde{y}; x_p + \xi_p) = \bar{L}(\tilde{y}; x_p) \; , \; S(x_p + \xi_p) = S(x_p)$$

$$\hat{L}(\tilde{y}) = \frac{\alpha}{12} \epsilon^{klm} V(\tilde{y}) H_k(\tilde{y}) \epsilon^{\mu\nu} \hat{\partial}_\mu H_l(\tilde{y}) \hat{\partial}_\nu H_m(\tilde{y}) + c.c.$$

Continuum Limit :

$$S = \frac{\alpha}{12} \int d^2x \epsilon^{klm} \epsilon^{\mu\nu} H_k(x) \partial_\mu H_l(x) \partial_\nu H_m(x) + c.c.$$

Lattice diffeomorphism transformation

$$\delta_p V(\tilde{y}) = \hat{\partial}_\mu \xi_p^\mu(\tilde{y}) V(\tilde{y})$$

$$\delta_p \hat{\partial}_\mu f(\tilde{y}) = -\hat{\partial}_\mu \xi_p^\nu(\tilde{y}) \hat{\partial}_\nu f(\tilde{y})$$

Unified theory in higher dimensions and energy momentum tensor

- Only spinors , no additional fields – no genuine source
- $J^{\mu}{}_{m}$: expectation values different from vielbein

  and incoherent fluctuations

- Can account for matter or radiation in effective four dimensional theory (including gauge fields as higher dimensional vielbein-components)

Gauge symmetries

Proposed action for lattice spinor gravity has also chiral SU(2) x SU(2) local gauge symmetry in continuum limit , acting on flavor indices.

Lattice action :
only global gauge symmetry realized

Gauge bosons, scalars …

from vielbein components
in higher dimensions
(Kaluza, Klein)

→

concentrate first on gravity