The Importance of Being Rigid

D6-Brane Model Building on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with Discrete Torsion

Wieland Staessens

In collaboration with G. Honecker and M. Ripka:

1209.3010 [hep-th]

PRiSMA

PRISMA Cluster of Excellence
Theoretical High Energy Physics,
JG Universität Mainz

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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Outline

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In type IIA on CY_3 orientifolds ($T^6/\mathbb{Z}_N \times \mathbb{Z}_M \times \Omega\mathcal{R}$):

- models with O6-planes and D6-planes wrapping Special Lagrangian 3-cycles
- Chiral matter arises at intersection points between 3-cycles
  $\Rightarrow$ chiral spectrum charged under $\prod_a U(N_a)$
  $\Rightarrow$ Topological intersection numbers encode chiral states and # generations
- Nice geometric picture + good perturbative control via CFT

Blumenhagen-Cvetič-Langacker-Shiu ('05); Blumenhagen-Körs-Lüst-Stieberger ('06); Ibañez-Uranga ('12); + other reviews
(see also Anastasopoulos' talk)

Drawbacks: presence of exotic matter
Adjoint matter naturally present for each $SU(N_a)$
$\rightarrow$ continuous breaking of gauge groups via displacement
Solution: wrap D-branes on rigid cycles

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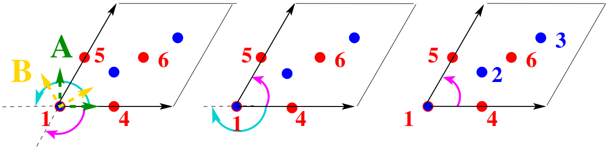
The orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

$\mathbb{Z}_2 \times \mathbb{Z}_6'$ action on $T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$ generated by

$$\begin{matrix} \vec{v} = \frac{1}{2}(1,-1,0), \\ \vec{w} = \frac{1}{6}(-2,1,1) \end{matrix} \; : \; z^k \longrightarrow e^{2\pi i\,(m v_k + n w_k)} z^k$$

Förste-Honecker [1010.6070]
Honecker [1109.3192]

Cristallographic action constrains shape of torus $\Rightarrow$ $\mathbb{C}$ structure moduli are fixed



$\mathbb{Z}_N$ subsectors	fixed points
$\mathbb{Z}_3$ on T^6	$< 1, 2, 3 >$
$3 \times \mathbb{Z}_2^{(i)}$ on $T^2_{(j)} \times T^2_{(k)}$	$< 1, 4, 5, 6 >$
$3 \times \mathbb{Z}_6$ on T^6	$< 1, 2, 3 >$

$\Rightarrow$ exceptional divisors $e^{(i)}_{xy}$
$x, y \in \{1, 4, 5, 6\}$

Type IIA on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: $\mathcal{N} = 2$ SUSY $\xrightarrow{\Omega\mathcal{R}}$ $\mathcal{N} = 1$ SUSY in 4dim

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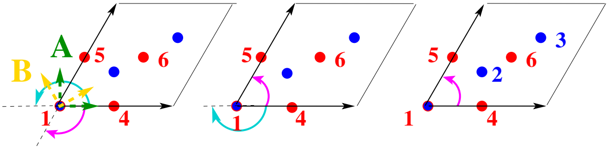

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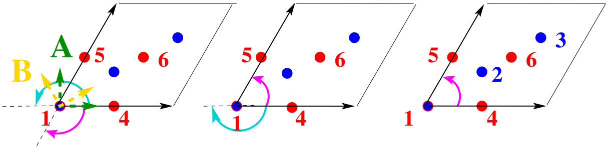

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The orientifold and discrete torsion

$\Omega\mathcal{R}$ projection acts on one-cycles as:

A $\pi_{2i-1} \longrightarrow \pi_{2i-1}, \quad \pi_{2i} \longrightarrow \pi_{2i-1} - \pi_{2i}$

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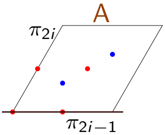

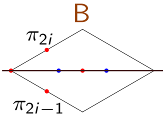

$4 \neq$ lattices: **AAA**, **AAB**, **ABB** and **BBB**

$\forall$ lattice: $4 \neq$ orbits O6-planes: $\Omega\mathcal{R}$-plane + $\Omega\mathcal{R}\mathbb{Z}_2^{(i)}$-planes $\quad (i = 1, 2, 3)$

Orbifolds $T^6/\mathbb{Z}_N \times \mathbb{Z}_M$ allow for **discrete torsion** η:

$\theta \in \mathbb{Z}_N$ acts with a phase η on $\mathbb{Z}_M$ twisted sector

$$\eta = e^{2\pi i\, n/\gcd(N,M)} \rightarrow \eta = \pm 1 \quad \text{for} \quad (N, M) = (2, 6')$$

Consistency: η related to RR-charges $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$ of O6-planes

$$\eta = \eta_{\Omega\mathcal{R}} \prod_{i=1}^{3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$$

Angelantonj et al. [hep-th/9911081]
Blumenhagen et al. [hep-th/0502095]

$\Rightarrow$ 1 **exotic** O6-plane $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$ & 3 ordinary O6-planes $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = 1$
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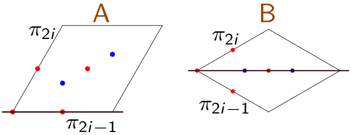

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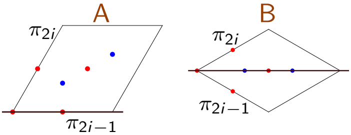

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Fractional cycles and chiral spectrum (I)

Canceling RR-charges of O6-planes requires D6-branes

(1) Bulk 3-cyle: $\sim \otimes_{i=1}^{3}(n^i \pi_{2i-1} + m^i \pi_{2i}), \qquad n^i, m^i \in \mathbb{Z}$

$\rightsquigarrow$ orbifold invariant basis $(\rho_1,\ \rho_2)$: $\Pi_a^{\text{bulk}} = X_a\, \rho_1 + Y_a\, \rho_2$
with $X_a(n_a^i, m_a^i) \in \mathbb{Z}$ and $Y_a(n_a^i, m_a^i) \in \mathbb{Z}$

(2) Exceptional 3-cycle: $\sim (n^i \pi_{2i-1} + m^i \pi_{2i}) \otimes e_{xy}^{(i)}$

$\Omega\mathcal{R} : e_{xy}^{(i)} \to -\eta_{(i)} e_{x'y'}^{(i)} \qquad$ with $\eta_{(i)} \equiv \eta_{\Omega\mathcal{R}}\, \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$

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$\cdots$ = discrete parameters at fixed points

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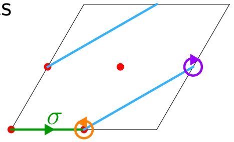

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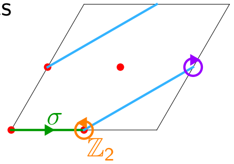

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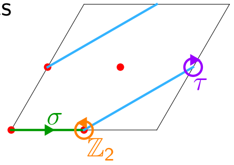

Fractional cycles and chiral spectrum (II)

(3) Fractional 3-cycle: $\Pi_a^{\rm frac} = \frac{1}{4}\left(\Pi_a^{\rm bulk} + \sum_{i=1}^{3} \Pi_a^{\mathbb{Z}_2^{(i)}}\right)$

$$\begin{aligned} \rightsquigarrow \chi^{(N_a,\overline{N}_b)} &\equiv \Pi_a^{\rm frac} \circ \Pi_b^{\rm frac} \\ &= \frac{1}{4}\left(X_a Y_b - Y_a X_b - \sum_{k=1}^{3}\sum_{\alpha=1}^{5}\left[x_{\alpha,a}^{(k)}\, y_{\alpha,b}^{(k)} - y_{\alpha,a}^{(k)}\, x_{\alpha,b}^{(k)}\right]\right) \end{aligned}$$

Chiral Spectrum:

sectors	rep.	net-chirality χ
$\sum_{k=0}^{2} a(\omega^k b)$	$(N_a, \overline{N_b})$	$\Pi_a \circ \Pi_b$
$\sum_{k=0}^{2} a(\omega^k b)'$	(N_a, N_b)	$\Pi_a \circ \Pi_{b'}$
$\sum_{k=0}^{2} (\omega^k a)(\omega^k a)'$	$\mathbf{Anti}_a$	$\frac{\Pi_a \circ \Pi_a' + \Pi_a \circ \Pi_{O6}}{2}$
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Separate sectors:
torus intersection numbers

$$\chi^{a(\omega^k b)} = -\frac{I_{a(\omega^k b)} + \sum_{i=1}^{3} I_{a(\omega^k b)}^{\mathbb{Z}_2^{(i)}}}{4}$$

$$\chi^{(N_a,\overline{N_b})} = \sum_{k=0}^{2} \chi^{a(\omega^k b)}$$

Note: non-chiral spectrum via β function coefficients

$$\begin{aligned} b_{SU(N)} &= N_a(-3 + \varphi^{\mathbf{Adj}_a}) + \frac{N_a}{2}(\varphi^{\mathbf{Sym}_a} + \varphi^{\mathbf{Anti}_a}) + (\varphi^{\mathbf{Sym}_a} - \varphi^{\mathbf{Anti}_a}) \\ &\quad + \sum_{b \neq a} \frac{N_b}{2}(\varphi^{(\mathbf{N}_a,\overline{\mathbf{N}}_b)} + \varphi^{(\mathbf{N}_a,\mathbf{N}_b)}) \end{aligned}$$

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$$\begin{aligned} b_{SU(N)} &= N_a(-3 + \varphi^{\mathbf{Adj}_a}) + \frac{N_a}{2}(\varphi^{\mathbf{Sym}_a} + \varphi^{\mathbf{Anti}_a}) + (\varphi^{\mathbf{Sym}_a} - \varphi^{\mathbf{Anti}_a}) \\ &\quad + \sum_{b \neq a} \frac{N_b}{2}(\varphi^{(\mathbf{N}_a,\overline{\mathbf{N}}_b)} + \varphi^{(\mathbf{N}_a,\mathbf{N}_b)}) \end{aligned}$$

Fractional cycles and chiral spectrum (II)

(3) Fractional 3-cycle: $\Pi_a^{\rm frac} = \frac{1}{4}\left(\Pi_a^{\rm bulk} + \sum_{i=1}^3 \Pi_a^{\mathbb{Z}_2^{(i)}}\right)$

$$\rightsquigarrow \chi^{(N_a,\overline{N}_b)} \equiv \Pi_a^{\rm frac} \circ \Pi_b^{\rm frac}$$

$$= \frac{1}{4}\left(X_a Y_b - Y_a X_b - \sum_{k=1}^{3}\sum_{\alpha=1}^{5}\left[x_{\alpha,a}^{(k)}\, y_{\alpha,b}^{(k)} - y_{\alpha,a}^{(k)}\, x_{\alpha,b}^{(k)}\right]\right)$$

Chiral Spectrum:

sectors	rep.	net-chirality χ
$\sum_{k=0}^2 a(\omega^k b)$	$(N_a, \overline{N}_b)$	$\Pi_a \circ \Pi_b$
$\sum_{k=0}^2 a(\omega^k b)'$	(N_a, N_b)	$\Pi_a \circ \Pi_{b'}$
$\sum_{k=0}^2 (\omega^k a)(\omega^k a)'$	$\textbf{Anti}_a$	$\frac{\Pi_a \circ \Pi_a' + \Pi_a \circ \Pi_{O6}}{2}$
	$\textbf{Sym}_a$	$\frac{\Pi_a \circ \Pi_a' - \Pi_a \circ \Pi_{O6}}{2}$

Separate sectors:
torus intersection numbers

$$\chi^{a(\omega^k b)} = -\frac{I_{a(\omega^k b)} + \sum_{i=1}^3 I_{a(\omega^k b)}^{\mathbb{Z}_2^{(i)}}}{4}$$

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$$b_{SU(N)} = N_a(-3 + \varphi^{\textbf{Adj}_a}) + \frac{N_a}{2}(\varphi^{\textbf{Sym}_a} + \varphi^{\textbf{Anti}_a}) + (\varphi^{\textbf{Sym}_a} - \varphi^{\textbf{Anti}_a})$$

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Pairwise identification between the lattices

Honecker-Ripka-Staessens [1209.3010]

Pairwise identification: **AAA** $\leftrightarrow$ **ABB** and **AAB** $\leftrightarrow$ **BBB**

- number of massless closed string states
- global consistency conditions (SUSY, RR tadpoles, etc.)
- intersection numbers + massless open string spectra
- string 1-loop amplitudes without operator insertions

boils down to

AAA	$\longleftrightarrow$	**ABB**
$(\eta_{(1)}, \eta_{(2)}, \eta_{(3)})$	$\leftrightarrow$	$(\eta_{(1)}, -\eta_{(2)}, -\eta_{(3)})$
$(n_a^1, m_a^1; n_a^2, m_a^2; n_a^3, m_a^3)$	$\leftrightarrow$	$(n_a^1 + m_a^1, -n_a^1; n_a^2, m_a^2; n_a^3, m_a^3)$
(X_a, Y_a)	$\leftrightarrow$	$(X_a + Y_a, -X_a)$
$(x^i_{\alpha,a}, y^i_{\alpha,a})$	$\leftrightarrow$	$(\bar{x}^i_\alpha(x_{\alpha,a}, y_{\alpha,a}), \bar{y}^i_\alpha(x_{\alpha,a}, y_{\alpha,a}))$
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Analogous relations for **AAB** $\leftrightarrow$ **BBB**

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Analogous relations for **AAB** $\leftrightarrow$ **BBB**

Towards model building

$\Rightarrow$ only two inequivalent lattices: **AAA** and **BBB**

Bulk RR tadpole cancellation conditions:

AAA $\sum_a N_a (2 X_a + Y_a) = 4 \left(\eta_{\Omega\mathcal{R}} + 3 \sum_{i=1}^{3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} \right)$

BBB $\sum_a N_a Y_a = 4 \left(3 \eta_{\Omega\mathcal{R}} + \sum_{i=1}^{3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} \right)$

SUSY conditions for bulk cycles:

AAA $Y_a = 0, \quad 2 X_a + Y_a > 0$

BBB $2 X_a + Y_a = 0, \quad Y_a > 0$

$$\text{rank}_{\textbf{AAA}} \leq \begin{cases} 16 & \eta_{\Omega\mathcal{R}} = -1 \\ 8 & \text{else} \end{cases} \qquad \text{rank}_{\textbf{BBB}} \leq \begin{cases} 8 & \text{else} \\ 0 & \eta_{\Omega\mathcal{R}} = -1 \end{cases}$$

$\Rightarrow$ large 'hidden' gauge groups possible on **AAA** with $\eta_{\Omega\mathcal{R}} = -1$

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Outline

Motivation

Introducing $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6' \times \Omega\mathcal{R}$

D6-brane model building

Pati-Salam models

Conclusions and Prospects

Rigid D6-Branes

Fractional cycles are stuck at $\mathbb{Z}_2^{(i)}$ fixed points

but $a(\omega^k a)_{k=1,2}$ sectors $\longrightarrow$ new adjoint multiplets

$$\chi^{a(\omega a)} = \frac{I_{a(\omega a)} + \sum_{i=1}^{3} I_{a(\omega a)}^{\mathbb{Z}_2^{(i)}}}{4} = -\chi^{a(\omega^2 a)}$$

rigid cycle: $\chi^{a(\omega^k a)} = 0, \quad k \in \{1,2\}$

$$\Rightarrow 1 + \sum_{i<j} \frac{1}{p_i p_j} = 0 \quad \text{with} \quad p_i \equiv (-1)^{\sigma_a^i \tau_a^i} \frac{L_a^{(i)}}{r_i} \in \mathbb{Z}_{\text{odd}}$$

rigidness depends on:

(i) lengths $L_a^{(i)}$ of factorisable 3-cycle

(ii) parameter combinations $\sigma_a^i \tau_a^i$

(iii) NOT on $\mathbb{Z}_2^{(i)}$ eigenvalues

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On AAA $\rightsquigarrow \exists$ rigid cycles for $L_a^{(i)} = r_i, \forall i$

(1) $a \upuparrows \Omega\mathcal{R}$: (1,0;1,0;1,0)

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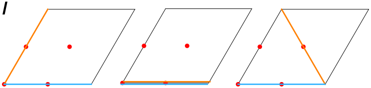

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Taking all discrete parameters into account: for each cycle

- 144 combinations: RIGID
- 108 combinations: 1 **Adj**

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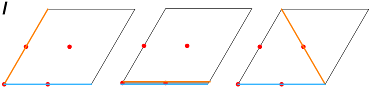

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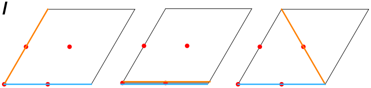

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Symmetric and Antisymmetric matter

On this orbifold: $\chi^{\textbf{Sym}} = \chi^{\textbf{Anti}}$

$\Rightarrow$ $SU(5)$ GUT excluded

$\Rightarrow$ r.h. quarks cannot arise as chiral antisymmetrics

3 $\neq$ situations have to be distinguished:

(i) $\phi^{(i)}_{(\omega^k a)(\omega^k a)'} \neq 0 \quad \forall i$

- depends on choice of exotic O6-plane
- independent of discrete parameters σ^i, τ^i

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e.g. a with $\vec{\phi}_{a\,\Omega\mathcal{R}} = \pi(\frac{1}{3_i}, 0_j, -\frac{1}{3_k})$: (0,1;1,0;1,-1) on **AAA**

sector	$\vec{\phi}_{(\omega^k a)(\omega^k a)'}$	$\#_{\textbf{Sym}} + \#_{\textbf{Anti}}$	$\#_{\textbf{Sym}} - \#_{\textbf{Anti}}$
aa'	$\pi(\frac{1}{3}, 0, -\frac{1}{3})$	$\frac{1}{2}(1+\eta_{(2)})$	$-\frac{\eta_{\Omega\mathcal{R}}}{2}(-1)^{\sigma_a^2\tau_a^2}(1+\eta_{(2)})$
$(\omega a)(\omega a)'$	$\pi(-\frac{1}{3}, \frac{1}{3}, 0)$	$\frac{1}{2}(1+\eta_{(3)})$	$-\frac{\eta_{\Omega\mathcal{R}}}{2}(-1)^{\sigma_a^3\tau_a^3}(1+\eta_{(3)})$
$(\omega^2 a)(\omega^2 a)'$	$\pi(0, -\frac{1}{3}, \frac{1}{3})$	$\frac{1}{2}(1+\eta_{(1)})$	$-\frac{\eta_{\Omega\mathcal{R}}}{2}(-1)^{\sigma_a^1\tau_a^1}(1+\eta_{(1)})$

- all sectors via (ii)
- If $\eta_{\Omega\mathcal{R}} = -1$ ($\eta_{(i)} = -1$): No symmetrics and no antisymmetrics
- If $\eta_{\Omega\mathcal{R}} = +1$: $\exists$ symmetrics and/or antisymmetrics
  and combinations $(\sigma^i\tau^i)$ determine representation

All other cycles on **AAA** come with symmetrics and/or antisymmetrics

Symmetric and Antisymmetric matter

e.g. a with $\vec{\phi}_{a\,\Omega\mathcal{R}} = \pi(\frac{1}{3_i}, 0_j, -\frac{1}{3_k})$: (0,1;1,0;1,-1) on **AAA**

sector	$\vec{\phi}_{(\omega^k a)(\omega^k a)'}$	$\#_{\text{Sym}} + \#_{\text{Anti}}$	$\#_{\text{Sym}} - \#_{\text{Anti}}$
aa'	$\pi(\frac{1}{3}, 0, -\frac{1}{3})$	$\frac{1}{2}\left(1+\eta_{(2)}\right)$	$-\frac{\eta_{\Omega\mathcal{R}}}{2}(-1)^{\sigma_a^2\tau_a^2}\left(1+\eta_{(2)}\right)$
$(\omega a)(\omega a)'$	$\pi(-\frac{1}{3}, \frac{1}{3}, 0)$	$\frac{1}{2}\left(1+\eta_{(3)}\right)$	$-\frac{\eta_{\Omega\mathcal{R}}}{2}(-1)^{\sigma_a^3\tau_a^3}\left(1+\eta_{(3)}\right)$
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3 Quark Generations

Constraints for the QCD stack:

No adjoints
No (chiral) symmetrics } $\Rightarrow$ Rigid D6-branes (0,1;1,0;1,-1) on **AAA** with $\eta_{\Omega\mathcal{R}} = -1$

Imposing constraints for QCD stack
$+$ requiring 3 quark generations
$\Rightarrow$ $SU(2)_L$ stack is completely fixed

$SU(2)_L$ stack	3 generations	
(1,0;1,0;1,0) w. **Adj**	NO	
(1,0;1,0;1,0) w/out. **Adj**	NO	
(0,1;1,0;1,-1) w. **Adj**	NO	
(0,1;1,0;1,-1) w/out. **Adj**	YES	$\Rightarrow 2\times27$ comb.

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Outline

Pati-Salam model

Pati-Salam model:

(1) lepton $\sim$ 4th quark flavour $\Rightarrow SU(4)$: strong gauge group

(2) left-right symmetric: $SU(2)_L \times SU(2)_R$ + right-handed neutrinos

5-stack Pati-Salam model: $SU(4) \times SU(2)_L \times SU(2)_R \times SU(2) \times SU(2)$

Chiral w.r.t. massive $U(1)^5$

SM:
$$(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + 2\,(\mathbf{4}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{1})$$
$$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1}) + 2\,(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1})$$
$$(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1}) \rightarrow 1 \text{ Higgs}$$

Other:
$$(\mathbf{1}, \mathbf{2}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + 3(\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \overline{\mathbf{2}})$$
$$(\mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{2}, \mathbf{1}) + 3(\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})$$

Non-chiral:
$$2\left[(\mathbf{4}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + h.c.\right] + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{4}_{\mathbf{Adj}}, \mathbf{1})$$
$$2\,[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{3}_{\mathbf{S}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}_{\mathbf{A}}, \mathbf{1}) + h.c.]$$
$$[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{3}_{\mathbf{S}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}_{\mathbf{A}}) + h.c.]$$
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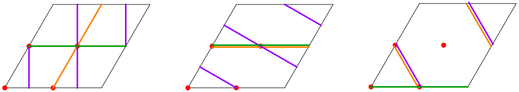

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$(1, 2, \overline{2}; 1, 1) \rightarrow 1$ Higgs

Other:
$(1, 2, 1; \overline{2}, 1) + 3(1, \overline{2}, 1; \overline{2}, 1) + (1, \overline{2}, 1; 1, \overline{2})$
$(1, 1, \overline{2}; 2, 1) + 3(1, 1, 2; 2, 1) + (1, 1, 2; 1, 2)$

Non-chiral:
$2\left[(4, 1, 1; \overline{2}, 1) + h.c.\right] + (1, 1, 1; 4_{\mathbf{Adj}}, 1)$
$2\left[(1, 1, 1; 3_{\mathbf{S}}, 1) + (1, 1, 1; 1_{\mathbf{A}}, 1) + h.c.\right]$
$\left[(1, 1, 1; 1, 3_{\mathbf{S}}) + (1, 1, 1; 1, 1_{\mathbf{A}}) + h.c.\right]$
$\left[(1, 1, 1; 2, 2) + h.c.\right]$

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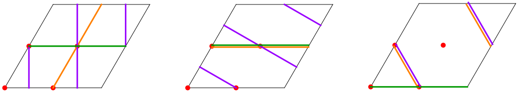

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Non-chiral
$$2\left[(\mathbf{4}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + h.c.\right] + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{4}_{\mathbf{Adj}}, \mathbf{1})$$
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$$[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{3}_{\mathbf{S}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}_{\mathbf{A}}) + h.c.]$$
$$[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{2}) + h.c.]$$

Outline

Conclusions and Prospects

Conclusions

- lattice identification $\Rightarrow$ only 2 inequivalent lattices **AAA** & **BBB**
- constraining exotic matter charged under QCD stack
  $+$ 3 quark generations constrains also $SU(2)_L$ stack drastically
- Global Pati-Salam models
- not covered here: local MSSM and LR symmetric
- not covered here: Yukawa couplings and other 3-point couplings

Prospects

- Other orbifolds: $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$

Ευχαριστώ πολύ

Gauge Groups Enhancements

Enhancement: $U(N_a) \to USp(2N_a) \Big/ SO(N_a) \Leftrightarrow \Omega\mathcal{R}(\Pi_a) = \Pi_a$

Useful for model building: $SU(2) \sim USp(2)$

$\Omega\mathcal{R}(\Pi_a) = \Pi_a$ can be written as topological conditions:

- bulk cycle $\upuparrows$ or $\perp$ exotic O6
- specific combinations of $(\vec{\sigma}, \vec{\tau})$ $\rightarrowtail$ $\eta_{(i)} \stackrel{!}{=} -(-)^{\sigma^j\tau^j + \sigma^k\tau^k}$

e.g. $a \upuparrows \Omega\mathcal{R}$ on **AAA**

$$\eta_{\Omega\mathcal{R}} = -1 \qquad \begin{array}{ll} \sigma^1\tau^1 = \sigma^2\tau^2 = \sigma^3\tau^3 = 0 & \rightsquigarrow USp(2N_a) \\ \sigma^1\tau^1 = \sigma^2\tau^2 = \sigma^3\tau^3 = 1 & \rightsquigarrow SO(2N_a) \end{array}$$

NOT RIGID

$$\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1 \qquad \begin{array}{ll} \sigma^i\tau^i = 1, \sigma^j\tau^j = \sigma^k\tau^k = 0 & \rightsquigarrow USp(2N_a) \\ \sigma^i\tau^i = 0, \sigma^j\tau^j = \sigma^k\tau^k = 1 & \rightsquigarrow SO(2N_a) \end{array}$$

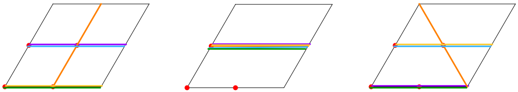
Note: for untilted (square) tori $\Omega\mathcal{R}$ invariance when $\upuparrows$ exotic O6 and $\forall\, \sigma^i\tau^i$ $SU(N_a) \to USp(2N_a)$, e.g. $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

Blumenhagen-Cvetič-Marchesano-Shiu [hep-th/0502095]

Pati-Salam model 2

6-stack Pati-Salam:

$SU(4) \times SU(2)_L \times SU(2)_R \times SU(4)_d \times SU(2)_e \times SU(2)_f$



Chiral w.r.t. massive $U(1)^6$

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$$2\,(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{4}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1})$$
$$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + 2\,(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1}, \mathbf{1})$$
$$2\,(\mathbf{1}, \mathbf{2}, \mathbf{2}; \mathbf{1}, \mathbf{1}, \mathbf{1}) \rightarrow 2 \text{ Higgses}$$

Other:

$$(\mathbf{1}, \mathbf{2}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \overline{\mathbf{2}})$$
$$(\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{2}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}, \mathbf{1})$$
$$(\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1}, \overline{\mathbf{2}})$$

Non-chiral

$$2\,[(\mathbf{6_A}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + h.c.] + 2\,[(\mathbf{1}, \mathbf{1}\,\mathbf{1}; \mathbf{6_A}, \mathbf{1}, \mathbf{1}) + h.c.]$$
$$2\,[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1_A}, \mathbf{1}) + h.c.] + 2\,[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1_A}) + h.c.]$$
$$\left[(\mathbf{4}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}) + h.c.\right] + [(\mathbf{4}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2}, \mathbf{1}) + h.c.]$$
$$\left[(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + h.c.\right] + \left[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \overline{\mathbf{2}}, \mathbf{1}) + h.c.\right]$$
$$[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{1}, \mathbf{2}) + h.c.] + \left[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}) + h.c.\right]$$