# Dimension Six Terms in the Standard Model Lagrangian 

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1 Introduction

- Effective theories
- Structure of the Standard Model

2 Reasoning scheme

3 Basis of invariant effective operators

4 Comparison with "Effective lagrangian analysis of new interactions and flavour conservation" by Buchmüller, Wyler (1986)

## Effective theories

## Example

Standard Model $\rightarrow$ Extension

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Standard Model $\rightarrow$ Extension
But how does Extension correct Standard Model interactions in low-energy processes?

Situation:
maximal momentum of particles $k \ll \Lambda$ scale of masses of heavy fields
$\rightarrow$ no explicit presence of heavy fields in the theory
$\rightarrow$ Appelquist-Carazzone decoupling theorem

$$
\mathcal{L}=\mathcal{L}_{S M}^{(4)}+\frac{1}{\Lambda} \sum_{i} c_{i}^{(5)} \mathcal{O}_{i}^{(5)}+\frac{1}{\Lambda^{2}} \sum_{i} c_{i}^{(6)} \mathcal{O}_{i}^{(6)}+O\left(\frac{1}{\Lambda^{3}}\right)
$$

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When the underlying theory is not yet known:

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When the underlying theory is known (e.g. it is SM) - effective theories give us calculational tools, that i.a. extend the validity of perturbation theory.

## Classifications of effective operators

$$
\mathcal{L}=\mathcal{L}_{S M}^{(4)}+\frac{1}{\Lambda} \sum_{i} c_{i}^{(5)} \mathcal{O}_{i}^{(5)}+\frac{1}{\Lambda^{2}} \sum_{i} c_{i}^{(6)} \mathcal{O}_{i}^{(6)}+O\left(\frac{1}{\Lambda^{3}}\right)
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Constraints:

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- Dependencies through EOM (H. D. Politzer, 1980)


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- C. N. Leung, S. T. Love and S. Rao (1986) - 106 operators, no reduction using EOM
- W. Buchmüller and D. Wyler (1986) applied EOM, getting 80 operators.
$\rightarrow$ but in fact only 59 operators are independent.


## SM - gauge group representations structure

| Field | representation (dimension) |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{SU}(2)$ | hypercharge <br> $\mathrm{U}(1)$ |  |
| $G_{\mu}$ | 8 | 1 | 0 |
| $W_{\mu}$ | 1 | 3 | 0 |
| $B_{\mu}$ | 1 | 1 | 0 |
| $q$ | 3 | 2 | $\frac{1}{6}$ |
| $u$ | 3 | 1 | $\frac{2}{3}$ |
| $d$ | 3 | 1 | $-\frac{1}{3}$ |
| $I$ | 1 | 2 | $-\frac{1}{2}$ |
| $e$ | 1 | 1 | -1 |
| $\varphi$ | 1 | 2 | $\frac{1}{2}$ |


| Type | vector $V_{\mu}$ | tensor $X_{\mu \nu}$ | spinor $\psi$ | skalar $\varphi$ |
| :--- | :---: | :---: | :---: | :---: |
| Dimension | $(G e V)^{1}$ | $(G e V)^{2}$ | $(G e V)^{\frac{3}{2}}$ | $(G e V)^{1}$ |
| Object | $D_{\mu}$ | $W_{\mu \nu}, G_{\mu \nu}, B_{\mu \nu}$ | $q, I, u, d, e$ | $\varphi$ |

- for $\operatorname{SU}(3)$

$$
G_{\mu \nu}^{A}=\partial_{\mu} G_{\nu}^{B}-\partial_{\nu} G_{\mu}^{C}-g_{s} f^{A B C} G_{\mu}^{B} G_{\nu}^{C}
$$

- for $\operatorname{SU}(2)$

$$
W_{\mu \nu}^{\prime}=\partial_{\mu} W_{\nu}^{\prime}-\partial_{\nu} W_{\mu}^{\prime}-g \varepsilon^{I J K} W_{\mu}^{J} W_{\nu}^{K}
$$

- for $\mathrm{U}(1)$

$$
B_{\mu \nu}=\partial_{\mu} B_{\nu}-\partial_{\nu} B_{\mu}
$$

1. Description in terms of matter fields $(\varphi, \psi)$, field strength tensors $X_{\mu \nu}$ and covariant derivatives $D_{\mu}$. Dimensional analysis.

## Reasoning scheme

1. Description in terms of matter fields $(\varphi, \psi)$, field strength tensors $X_{\mu \nu}$ and covariant derivatives $D_{\mu}$. Dimensional analysis. e.g. dim-6 expressions containing both fermionic and bosonic fields: $\psi \psi X D, \psi \psi X \varphi, \psi \psi \varphi \varphi \varphi, \psi \psi \varphi \varphi D, \psi \psi \varphi D D, \psi \psi D D D$
2. Gauge and Lorentz symmetry.
3. Description in terms of matter fields $(\varphi, \psi)$, field strength tensors $X_{\mu \nu}$ and covariant derivatives $D_{\mu}$. Dimensional analysis.
4. Gauge and Lorentz symmetry.

## Reasoning scheme

1. Description in terms of matter fields $(\varphi, \psi)$, field strength tensors $X_{\mu, 1}$ and covariant derivatives $D_{\mu \prime}$. Dimensional analysis.
2. Gauge and Lorentz symmetry.
e.g. $\psi \psi \varphi D D$ :
many possible choices of $\psi$ - the only singlet in $\hat{2}_{S U(2)} \otimes \hat{2}_{S U(2)}$ hypercharge conservation

$$
\left(q^{\dagger} \varepsilon \varphi^{*}\right) u, \quad\left(q^{\dagger} \varphi\right) d, \quad\left(I^{\dagger} \varphi\right) e, \quad+\text { h.c. }
$$

3. Reduction of the set of operators using algebraic properties and SM EOM:

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$$

Lorentz structure contains 2 singlets:

$$
\begin{aligned}
\left(0, \frac{1}{2}\right) \otimes\left(0, \frac{1}{2}\right) \otimes\left(\frac{1}{2}, \frac{1}{2}\right) \otimes\left(\frac{1}{2}, \frac{1}{2}\right)= & (0,0) \oplus(0,0) \oplus(1,0) \oplus(2,0) \\
& \oplus(1,1) \oplus(0,1) \oplus(1,1) \oplus(2,1)
\end{aligned}
$$

Reduction of the set of operators using algebraic properties and SM EOM:

## Reasoning scheme

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$$

2 independent Lorentz invariants (for each):

$$
\bar{\psi}_{L} \psi_{R} \varphi D_{\mu} D^{\mu} \quad \bar{\psi}_{L} \sigma_{\mu \nu} \psi_{R} \varphi D^{\mu} D^{\nu}
$$

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## Reasoning scheme

1. Description in terms of matter fields $(\varphi, \psi)$, field strength tensors $X_{I I,}$ and covariant derivatives $D_{\mu L}$. Dimensional analysis.
2. Gauge and Lorentz symmetry.
3. Reduction of the set of operators using algebraic properties and SM EOM:
We have (omitting full div) the following operators:

$$
\begin{gather*}
\left(\bar{\psi}_{L} \sigma_{\mu \nu} \psi_{R}\right)\left(D^{\mu} D^{\nu} \varphi\right)  \tag{1}\\
\left(\bar{\psi}_{L} \sigma_{\mu \nu} D^{\mu} D^{\nu} \psi_{R}\right) \varphi  \tag{2}\\
\left(\bar{\psi}_{L} \sigma_{\mu \nu} D^{\mu} \psi_{R}\right)\left(D^{\nu} \varphi\right)  \tag{3}\\
\left(\bar{\psi}_{L} D_{\mu} D^{\mu} \psi_{R}\right) \varphi  \tag{4}\\
\left(\bar{\psi}_{L} D_{\mu} \psi_{R}\right)\left(D^{\mu} \varphi\right)  \tag{5}\\
\left(\bar{\psi}_{L} \psi_{R}\right)\left(D_{\mu} D^{\mu} \varphi\right) \tag{6}
\end{gather*}
$$

## Reasoning scheme

1. Description in terms of matter fields $(\varphi, \psi)$, field strength tensors $X_{\mu \prime \prime}$ and covariant derivatives $D_{\mu \prime}$. Dimensional analysis.
2. Gauge and Lorentz symmetry.
3. Reduction of the set of operators using algebraic properties and SM EOM:
Using EOM for $\varphi$ :

$$
-\left(D_{\mu} D^{\mu} \varphi\right)+m^{2} \varphi-\lambda \varphi\left(\varphi^{\dagger} \varphi\right)-\Gamma_{e}^{\dagger} / \bar{e}+\Gamma_{u} \varepsilon \bar{q} u-\Gamma_{d}^{\dagger} q \bar{d}=0
$$

we get

$$
\left(\bar{\psi}_{L} \psi_{R}\right)\left(D_{\mu} D^{\mu} \varphi\right)=\psi \psi \varphi+\psi \psi \varphi \varphi \varphi+\psi \psi \psi \psi
$$



## Bosonic invariant operators

| $X^{3}$ |  | $\varphi^{6}$ and $\varphi^{4} D^{2}$ |  |
| :---: | :---: | :---: | :---: |
| $Q_{G}$ | $f^{A B C} G_{\mu}^{A \nu} G_{\nu}^{B \rho} G_{\rho}^{C \mu}$ | $Q_{\varphi}$ | $\left(\varphi^{\dagger} \varphi\right)^{3}$ |
| $Q_{\widetilde{G}}$ | $f^{A B C} \widetilde{G}_{\mu}^{A \nu} G_{\nu}^{B \rho} G_{\rho}^{C \mu}$ | $Q_{\varphi \square}$ | $\left(\varphi^{\dagger} \varphi\right) \square\left(\varphi^{\dagger} \varphi\right)$ |
| $Q_{W}$ | $\varepsilon^{I J K} W_{\mu}^{I \nu} W_{\nu}^{J \rho} W_{\rho}^{K \mu}$ | $Q_{\varphi D}$ | $\left(\varphi^{\dagger} D^{\mu} \varphi\right)^{\star}\left(\varphi^{\dagger} D_{\mu} \varphi\right)$ |
| $Q_{\widetilde{W}}$ | $\varepsilon^{I J K} \widetilde{W}_{\mu}^{I \nu} W_{\nu}^{J \rho} W_{\rho}^{K \mu}$ |  |  |

## Bosonic invariant operators

| $X^{2} \varphi^{2}$ |  |
| :--- | :--- |
| $Q_{\varphi G}$ | $\varphi^{\dagger} \varphi G_{\mu \nu}^{A} G^{A \mu \nu}$ |
| $Q_{\varphi \widetilde{G}}$ | $\varphi^{\dagger} \varphi \widetilde{G}_{\mu \nu}^{A} G^{A \mu \nu}$ |
| $Q_{\varphi W}$ | $\varphi^{\dagger} \varphi W_{\mu \nu}^{I} W^{I \mu \nu}$ |
| $Q_{\varphi \widetilde{W}}$ | $\varphi^{\dagger} \varphi \widetilde{W}_{\mu \nu}^{I} W^{I \mu \nu}$ |
| $Q_{\varphi B}$ | $\varphi^{\dagger} \varphi B_{\mu \nu} B^{\mu \nu}$ |
| $Q_{\varphi \widetilde{B}}$ | $\varphi^{\dagger} \varphi \widetilde{B}_{\mu \nu} B^{\mu \nu}$ |
| $Q_{\varphi W B}$ | $\varphi^{\dagger} \tau^{I} \varphi W_{\mu \nu}^{I} B^{\mu \nu}$ |
| $Q_{\varphi \widetilde{W} B}$ | $\varphi^{\dagger} \tau^{I} \varphi \widetilde{W_{\mu \nu}^{I}} B^{\mu \nu}$ |

## Invariant operators with 2 fermions

|  |  |
| :--- | :--- |
| $Q^{2} \varphi^{3}$ |  |
| $Q_{u \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{l}_{p} e_{r} \varphi\right)$ |
| $Q_{d \varphi}$ | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{q}_{p} u_{r} \widetilde{\varphi}\right)$ |
|  | $\left(\varphi^{\dagger} \varphi\right)\left(\bar{q}_{p} d_{r} \varphi\right)$ |

( $\varphi^{2}$ singlet)(SM Yukawa couplings of $\varphi$ )

## Invariant operators with 2 fermions

| $\psi^{2} X \varphi$ |  | $\psi^{2} \varphi^{2} D$ |  |
| :---: | :---: | :---: | :---: |
| $Q_{e W}$ | $\left(\bar{l}_{p} \sigma^{\mu \nu} e_{r}\right) \tau^{I} \varphi W_{\mu \nu}^{I}$ | $Q_{\varphi l}^{(1)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}} \varphi\right)\left(\bar{l}_{p} \gamma^{\mu} l_{r}\right)$ |
| $Q_{e B}$ | $\left(\bar{l}_{p} \sigma^{\mu \nu} e_{r}\right) \varphi B_{\mu \nu}$ | $Q_{\varphi l}^{(3)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}^{I}} \varphi\right)\left(\bar{l}_{p} \tau^{I} \gamma^{\mu} l_{r}\right)$ |
| $Q_{u G}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} T^{A} u_{r}\right) \widetilde{\varphi} G_{\mu \nu}^{A}$ | $Q_{\varphi e}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}} \varphi\right)\left(\bar{e}_{p} \gamma^{\mu} e_{r}\right)$ |
| $Q_{u W}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} u_{r}\right) \tau^{I} \widetilde{\varphi} W_{\mu \nu}^{I}$ | $Q_{\varphi q}^{(1)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}} \varphi\right)\left(\bar{q}_{p} \gamma^{\mu} q_{r}\right)$ |
| $Q_{u B}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} u_{r}\right) \widetilde{\varphi} B_{\mu \nu}$ | $Q_{\varphi q}^{(3)}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{\left.D_{\mu}^{I} \varphi\right)\left(\bar{q}_{p} \tau^{I} \gamma^{\mu} q_{r}\right)}\right.$ |
| $Q_{d G}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} T^{A} d_{r}\right) \varphi G_{\mu \nu}^{A}$ | $Q_{\varphi u}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}} \varphi\right)\left(\bar{u}_{p} \gamma^{\mu} u_{r}\right)$ |
| $Q_{d W}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} d_{r}\right) \tau^{I} \varphi W_{\mu \nu}^{I}$ | $Q_{\varphi d}$ | $\left(\varphi^{\dagger} i \stackrel{\leftrightarrow}{D_{\mu}} \varphi\right)\left(\bar{d}_{p} \gamma^{\mu} d_{r}\right)$ |
| $Q_{d B}$ | $\left(\bar{q}_{p} \sigma^{\mu \nu} d_{r}\right) \varphi B_{\mu \nu}$ | $Q_{\varphi u d}$ | $i\left(\widetilde{\varphi}^{\dagger} D_{\mu} \varphi\right)\left(\bar{u}_{p} \gamma^{\mu} d_{r}\right)$ |

## Fermionic operators

## General structure:

$$
\left(\bar{\psi}_{1} \gamma_{\mu} \psi_{1}\right)\left(\bar{\psi}_{2} \gamma^{\mu} \psi_{2}\right) \quad\left(\bar{\psi}_{1} \gamma_{\mu} T^{\alpha} \psi_{1}\right)\left(\bar{\psi}_{2} \gamma^{\mu} T^{\alpha} \psi_{2}\right)
$$

| $\left(\bar{L}_{L} L\right)(\bar{L} L)$ |  | $\left(\bar{R}^{2} R\right)\left(\bar{R}^{2} R\right)$ |  | $\left(\bar{L}^{2} L\right)\left(\bar{R}^{2} R\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{l l}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{l}_{s} \gamma^{\mu} l_{t}\right)$ | $Q_{e e}$ | $\left(\bar{e}_{p} \gamma_{\mu} e_{r}\right)\left(\bar{e}_{s} \gamma^{\mu} e_{t}\right)$ | $Q_{l e}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{e}_{s} \gamma^{\mu} e_{t}\right)$ |
| $Q_{q q}^{(1)}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} q_{t}\right)$ | $Q_{u u}$ | $\left(\bar{u}_{p} \gamma_{\mu} u_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} u_{t}\right)$ | $Q_{l u}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} u_{t}\right)$ |
| $Q_{q q}^{(3)}$ | $\left(\bar{q}_{p} \gamma_{\mu} \tau^{I} q_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} \tau^{I} q_{t}\right)$ | $Q_{d d}$ | $\left(\bar{d}_{p} \gamma_{\mu} d_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ | $Q_{l d}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ |
| $Q_{l q}^{(1)}$ | $\left(\bar{l}_{p} \gamma_{\mu} l_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} q_{t}\right)$ | $Q_{e u}$ | $\left(\bar{e}_{p} \gamma_{\mu} e_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} u_{t}\right)$ | $Q_{q e}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{e}_{s} \gamma^{\mu} e_{t}\right)$ |
| $Q_{l q}^{(3)}$ | $\left(\bar{l}_{p} \gamma_{\mu} \tau^{I} l_{r}\right)\left(\bar{q}_{s} \gamma^{\mu} \tau^{I} q_{t}\right)$ | $Q_{e d}$ | $\left(\bar{e}_{p} \gamma_{\mu} e_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ | $Q_{q u}^{(1)}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} u_{t}\right)$ |
|  |  | $Q_{u d}^{(1)}$ | $\left(\bar{u}_{p} \gamma_{\mu} u_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ | $Q_{q u}^{(8)}$ | $\left(\bar{q}_{p} \gamma_{\mu} T^{A} q_{r}\right)\left(\bar{u}_{s} \gamma^{\mu} T^{A} u_{t}\right)$ |
|  |  | $Q_{u d}^{(8)}$ | $\left(\bar{u}_{p} \gamma_{\mu} T^{A} u_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} T^{A} d_{t}\right)$ | $Q_{q d}^{(1)}$ | $\left(\bar{q}_{p} \gamma_{\mu} q_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} d_{t}\right)$ |
|  |  |  | $Q_{q d}^{(8)}$ | $\left(\bar{q}_{p} \gamma_{\mu} T^{A} q_{r}\right)\left(\bar{d}_{s} \gamma^{\mu} T^{A} d_{t}\right)$ |  |

# $(\bar{L} R)(\bar{R} L)$ and $(\bar{L} R)(\bar{L} R)$ 

$Q_{l e d q}$
$\left({ }_{p}^{\bar{l}} e_{r}\right)\left(\bar{d}_{s} q_{t}^{j}\right)$
$Q_{q u q d}^{(1)}$
$\left(\bar{q}_{p}^{j} u_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} d_{t}\right)$
$Q_{q u q d}^{(8)}$
$\left(\bar{q}_{p}^{j} T^{A} u_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} T^{A} d_{t}\right)$
$Q_{\text {lequ }}^{(1)}$
$\left(\bar{l}_{p}^{j} e_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} u_{t}\right)$
$Q_{\text {lequ }}^{(3)}$
$\left(\bar{l}_{p}^{j} \sigma_{\mu \nu} e_{r}\right) \varepsilon_{j k}\left(\bar{q}_{s}^{k} \sigma^{\mu \nu} u_{t}\right)$

## Redundant operators

$$
\frac{1}{2} \partial_{\mu}\left(\varphi^{\dagger} \varphi\right) \partial^{\mu}\left(\varphi^{\dagger} \varphi\right)
$$

$$
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$$

$$
\begin{gathered}
\left(\bar{\psi}_{R} D_{\mu} \psi_{L}\right)\left(D^{\mu} \varphi\right) \\
{\left[\left(D_{\mu} \bar{\psi}_{R}\right) \psi_{L}\right]\left(D^{\mu} \varphi\right)} \\
\left(\bar{\psi} \gamma_{\mu} D_{\nu} \psi\right) X^{\mu \nu}
\end{gathered}
$$

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$$

$$
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\left(\bar{\psi}_{R} D_{\mu} \psi_{L}\right)\left(D^{\mu} \varphi\right) \\
{\left[\left(D_{\mu} \bar{\psi}_{R}\right) \psi_{L}\right]\left(D^{\mu} \varphi\right)} \\
\left(\bar{\psi} \gamma_{\mu} D_{\nu} \psi\right) X^{\mu \nu}
\end{gathered}
$$

$$
\begin{aligned}
& \left(\bar{T}_{p_{1}} \gamma_{\mu} T^{\prime} l_{p_{2}}\right)\left(\bar{T}_{p_{3}} \gamma^{\mu} T^{\prime} l_{p_{4}}\right. \\
& T_{i j}^{\prime} T_{k l}^{\prime}=\frac{1}{2} \delta_{i l} \delta_{k j}-\frac{1}{2 N} \delta_{i j} \delta_{k l}
\end{aligned}
$$

$$
\frac{1}{2} \partial_{\mu}\left(\varphi^{\dagger} \varphi\right) \partial^{\mu}\left(\varphi^{\dagger} \varphi\right)
$$

$$
\begin{gathered}
\left(\bar{\psi}_{R} D_{\mu} \psi_{L}\right)\left(D^{\mu} \varphi\right) \\
{\left[\left(D_{\mu} \bar{\psi}_{R}\right) \psi_{L}\right]\left(D^{\mu} \varphi\right)} \\
\left(\bar{\psi} \gamma_{\mu} D_{\nu} \psi\right) X^{\mu \nu}
\end{gathered}
$$

$$
\begin{aligned}
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& T_{i j}^{\prime} T_{k l}^{\prime}=\frac{1}{2} \delta_{i l} \delta_{k j}-\frac{1}{2 N} \delta_{i j} \delta_{k l}
\end{aligned}
$$

The absent one: $(\bar{q} e) \varepsilon(\bar{l} u)^{T}$

## Independence of listed operators

How do we know that these operators are finally independent?
Main arguments:

- field content
- EOM contain at least one/two derivatives


## Independence of listed operators

How do we know that these operators are finally independent?
Main arguments:

- field content
- EOM contain at least one/two derivatives

Why $(\bar{q} e) \varepsilon(\bar{l} u)^{T}$ must be on the list?

- completeness of classification
- field content


## Lepton/baryon nr violating invariant operators

$$
\left(I^{T} \tilde{\varphi}^{*}\right)\left(\tilde{\varphi}^{\dagger} l\right)
$$

| B-violating |  |
| :---: | :---: |
| $Q_{d u q}$ | $\varepsilon^{\alpha \beta \gamma} \varepsilon_{j k}\left[\left(d_{p}^{\alpha}\right)^{T} C u_{r}^{\beta}\right]\left[\left(q_{s}^{\gamma j}\right)^{T} C l_{t}^{k}\right]$ |
| $Q_{q q u}$ | $\varepsilon^{\alpha \beta \gamma} \varepsilon_{j k}\left[\left(q_{p}^{\alpha j}\right)^{T} C q_{r}^{\beta k}\right]\left[\left(u_{s}^{\gamma}\right)^{T} C e_{t}\right]$ |
| $Q_{q q q}^{(1)}$ | $\varepsilon^{\alpha \beta \gamma} \varepsilon_{j k} \varepsilon_{m n}\left[\left(q_{p}^{\alpha j}\right)^{T} C q_{r}^{\beta k}\right]\left[\left(q_{s}^{\gamma m}\right)^{T} C l_{t}^{n}\right]$ |
| $Q_{q q q}^{(3)}$ | $\varepsilon^{\alpha \beta \gamma}\left(\tau^{I} \varepsilon\right)_{j k}\left(\tau^{I} \varepsilon\right)_{m n}\left[\left(q_{p}^{\alpha j}\right)^{T} C q_{r}^{\beta k}\right]\left[\left(q_{s}^{\gamma m}\right)^{T} C l_{t}^{n}\right]$ |
| $Q_{d u u}$ | $\varepsilon^{\alpha \beta \gamma}\left[\left(d_{p}^{\alpha}\right)^{T} C u_{r}^{\beta}\right]\left[\left(u_{s}^{\gamma}\right)^{T} C e_{t}\right]$ |

list already given by L.F. Abbott and M. B. Wise (1980), which simplified those given by S. Weinberg (1976) and F. Wilczek and A. Zee (1979)

# The importance of classification 

How to discover new physics?

How to discover new physics?
Recent papers with redundant operators:

- J. A. Aguilar-Saavedra, "Single top quark production at LHC with anomalous Wtb couplings", Nucl. Phys. B804 (2008) 160;
- K. Agashe, R. Contino, "Composite Higgs-mediated flavor-changing neutral current", Phys. Rev. D 80, 075016 (2009);
- S. Kanemura, K. Tsumura, "Effects of the anomalous Higgs couplings on the Higgs boson production at the Large Hadron Collider ", Eur. Phys. J. C63 (2009) 11;
- Bohdan Grzadkowski, Mikołaj Misiak, Janusz Rosiek, MI "Dimension-Six Terms in the Standard Model Lagrangian" JHEP 1010:085,2010
- J. A. Aguilar-Saavedra
"Effective four-fermion operators in top physics: a roadmap" Nucl.Phys.B843:638-672,2011


## Questions



## Example: reduction of $\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)$

$$
\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \eta^{\nu} \psi\right)\left(D_{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \frac{1}{2}\left\{\gamma^{\nu}, \gamma^{\mu}\right\} \psi\right)\left(D_{\mu} \varphi\right)
$$

$$
=\frac{1}{2}\left(\bar{\psi} D_{\nu} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)+\frac{1}{2}\left(\bar{\psi} \gamma^{\mu} \underline{D \psi}\right)\left(D_{\mu} \varphi\right)
$$



$$
-\frac{1}{2}\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\nu} D_{\mu} \varphi\right)
$$

## Example: reduction of $\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)$

$$
\begin{gathered}
\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \eta^{\nu} \psi\right)\left(D_{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \frac{1}{2}\left\{\gamma^{\nu}, \gamma^{\mu}\right\} \psi\right)\left(D_{\mu} \varphi\right) \\
=\frac{1}{2}\left(\bar{\psi} D_{\nu} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)+\frac{1}{2}\left(\bar{\psi} \gamma^{\mu} \underline{D \psi}\right)\left(D_{\mu} \varphi\right)
\end{gathered}
$$

## Example: reduction of $\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)$

$$
\begin{aligned}
& \left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \eta^{\nu \mu} \psi\right)\left(D_{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \frac{1}{2}\left\{\gamma^{\mu}, \gamma^{\mu}\right\} \psi\right)\left(D_{\mu} \varphi\right) \\
& =\frac{1}{2}\left(\bar{\psi} D_{\nu} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)+\frac{1}{2}\left(\bar{\psi} \gamma^{\mu} \underline{D \psi}\right)\left(D_{\mu} \varphi\right) \\
& =\psi \psi \varphi \varphi D+\frac{1}{2} D_{\nu}\left[\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)\right]-\frac{1}{2}\left(\bar{\psi} \overleftarrow{\bar{D}} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)+ \\
& -\frac{1}{2}\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\nu} D_{\mu} \varphi\right)
\end{aligned}
$$

## Example: reduction of $\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)$

$$
\begin{aligned}
& \left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)=\left(\bar{\psi} D_{\nu} \frac{1}{2}\left\{\gamma^{\mu}, \gamma^{\mu}\right\} \psi\right)\left(D_{\mu} \varphi\right) \\
& =\frac{1}{2}\left(\bar{\psi} D_{\nu} \gamma^{\mu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)+\frac{1}{2}\left(\bar{\psi} \gamma^{\mu} D \psi\right)\left(D_{\mu} \varphi\right) \\
& =\psi \psi \varphi \varphi D+\frac{1}{2} D_{\nu}\left[\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)\right]-\frac{1}{2}\left(\bar{\psi} \overleftarrow{\bar{D}} \gamma^{\mu} \psi\right)\left(D_{\mu} \varphi\right)+ \\
& -\frac{1}{2}\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\nu} D_{\mu} \varphi\right)
\end{aligned}
$$

but

$$
\begin{equation*}
\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\nu} D_{\mu} \varphi\right)=\left(\bar{\psi}(-i) \sigma^{\nu \mu} \psi\right)\left(\sum_{k} X_{\nu \mu}^{k} \varphi\right)+(\bar{\psi} \psi)\left(\underline{D^{\mu} D_{\mu} \varphi}\right) \tag{SO}
\end{equation*}
$$

## Example: reduction of $\left(\bar{\psi} D_{\mu} \psi\right)\left(D^{\mu} \varphi\right)$


but

$$
\left(\bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi\right)\left(D_{\nu} D_{\mu} \varphi\right)=\left(\bar{\psi}(-i) \sigma^{\nu \mu} \psi\right)\left(\sum_{k} X_{\nu \mu}^{k} \varphi\right)+(\bar{\psi} \psi)\left(\underline{D^{\mu} D_{\mu} \varphi}\right)
$$

so
$\left(\bar{\psi}_{L} D_{\mu} \psi_{R}\right)\left(D^{\mu} \varphi\right)=\psi \psi \varphi \varphi D+\psi \psi X \varphi+\psi \psi \varphi+\psi \psi \varphi \varphi \varphi+\psi \psi \psi \psi$

