State sum models and the spectral action

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Outline

The spectral action

Induced Standard Model

State sum models

Standard model

Fermions: $\Psi = 8 \times 3$ Dirac spinors

Bosons: $d =$ gravitational Dirac

$A =$ gauge fields

$H =$ Higgs

Generalised Dirac operator

$$D = d + A + yH + m$$

$y =$ Yukawa mass matrix

$m =$ Majorana mass matrix

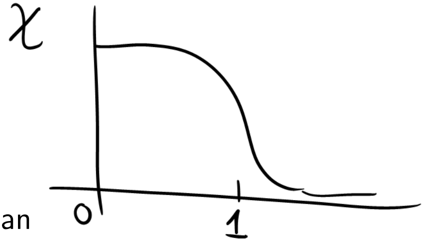
Fermionic action

$$S = \int \overline{\Psi} D \Psi \, \mathrm{d}V$$

Connes-Chamseddine spectral action

$$S_{CC} = \mathrm{Tr}\,\chi(D^2/c^2)$$

c = cutoff energy



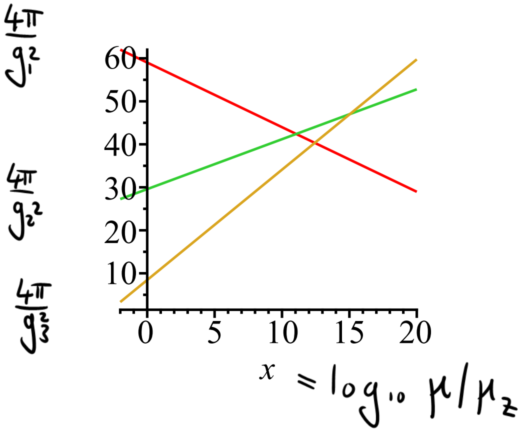
- Euclidean
- Spectral
- Asymptotics $\rightarrow$ Bosonic SM + gravity

Gauge couplings:

SU(3)	SU(2)	U(1)
g_3	g_2	g'

- $g_3 = g_2 = g_1 = \sqrt{\frac{5}{3}}g'$
- etc

Running couplings



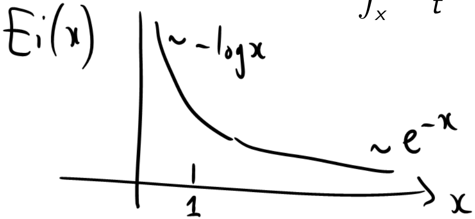
Fermion functional integral

$$\det D = e^{\frac{1}{2}\mathrm{Tr}\log D^2} = e^{-I}$$

- Cutoff energy c (quantum gravity)

Cutoff example (heat kernel): $\log \to -\mathrm{Ei}$

$$\mathrm{Ei}(x) = \int_x^\infty \frac{\mathrm{d}t}{t}\, e^{-t}$$



Hence induced bosonic action

$$I = \frac{1}{2}\mathrm{Tr}\,\mathrm{Ei}(D^2/c^2)$$

… as in Induced Gravity (Sakharov)

$-+++$ metric and N fermion fields. Cutoff $= c$.

If

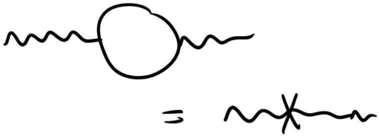
$$S = \int \left(\overline{\psi} D_c \psi - 2\Lambda_0\right) \mathrm{d}V$$

Integrating over fermion modes gives

$$I = \int \frac{-c^4 N}{32\pi^2} - 2\Lambda_0 + \frac{c^2 N}{192\pi^2} R + \text{etc.} \quad \mathrm{d}V$$

- Correct sign for R (MTW signs)
- Cosmological constant $\Lambda = \Lambda_0 + \frac{c^4 N}{64\pi^2}$
- Effective below fermion mass

Example: induced Yang-Mills term



- A gauge field
- Ψ Dirac fermion, mass $m \neq 0$

gives induced Yang-Mills term

$$\frac{1}{g^2} \int \mathrm{Tr} F^2 \, \mathrm{d}V.$$

Induced coupling constant

$\frac{1}{2}$ non-ab

$$\frac{1}{\alpha} = \frac{4\pi}{g^2} = \frac{2}{3\pi} \; T(R) \; \log c/m.$$

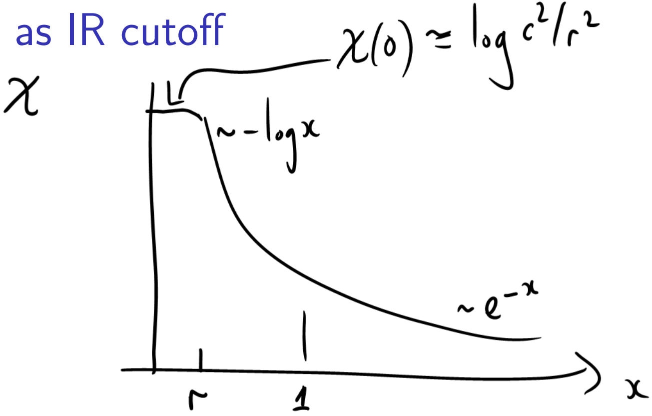
Yang Mills couplings

$$\frac{1}{\alpha} = \frac{1}{3\pi} \sum_{\text{Weyl fermions}} T(R) \, \log c/m.$$

Compare with Connes-Chamseddine

$$\frac{1}{\alpha_{CC}} = \frac{1}{6\pi} \sum_{\text{Weyl fermions}} T(R) \, \chi(0)$$

CC as IR cutoff



$$\frac{1}{\alpha_{CC}} = \frac{1}{6\pi} \sum_{\text{Weyl}} T(R)\, \chi(0) = \frac{1}{3\pi} \sum_{\text{Weyl}} T(R)\ \log c/r$$

- Scaling $c \to \infty$, $r \to \infty$, eventually $r > m$

Partition function

$$Z = \int \mathrm{d}D \, \mathrm{d}\Psi \, \mathrm{d}\overline{\Psi} \; e^{iS(D,\psi)}$$

as ‘usual’. (State sum model?)

Below fermion masses, fermions can be removed, renormalising the bosonic couplings (fermion decoupling theorem)

Induced standard model (speculative)

$$S = \int (\overline{\Psi} D \Psi - 2\Lambda_0) \, \mathrm{d}V$$

- Bosonic SM+gravity action induced...

Possible scenarios

- Extra generation of heavy fermions (e.g. S_{CC})
- RH neutrino
- New physics

State sum models

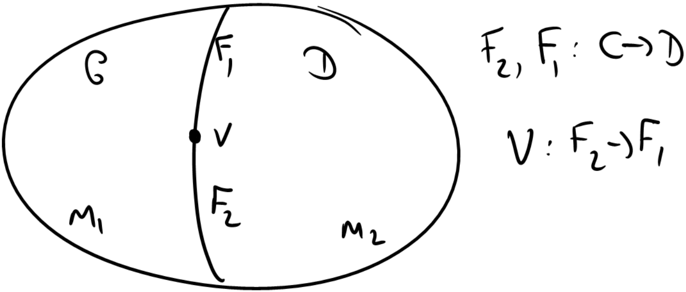
- Discrete functional integrals on a triangulated manifold
- Input: gauge group (or category $\mathcal{C}$)
- Optional input: data for a local observable
- Output: partition function $Z \in \mathbb{C}$

Examples from physics

- lattice gauge theory
- 2d BF
- 3d quantum gravity
- 4d quantum gravity models

Geometry - matter coupling (+C. Meusburger)

- Matter as defects in a state sum model



$$Z = \langle F(Z(M_1)), Z(M_2) \rangle$$

State sum models - project (speculative)

- Couple 4d state sum model to fermionic matter
- Induce gravity (+SM)
- Compare lowest-order induced action with existing gravity models