UNIVERSITAT DE BARCELONA

Institut de Ciències del Cosmos

# The Cosmological Constant Problem and the Relaxation of the Vacuum Energy


**Joan Solà**

sola@ecm.ub.es

**HEP Group**
**Departament d'Estructura i Constituents de la Matèria (ECM)**
**Institut de Ciències del Cosmos, Univ. Barcelona**


**Corfu Summer Institute**, August 29-September 5, 2010

# Guidelines of the Talk

Dark Energy and the CC problems

The fine-tuning problem in QFT

Dynamical CC term in Einstein's equations

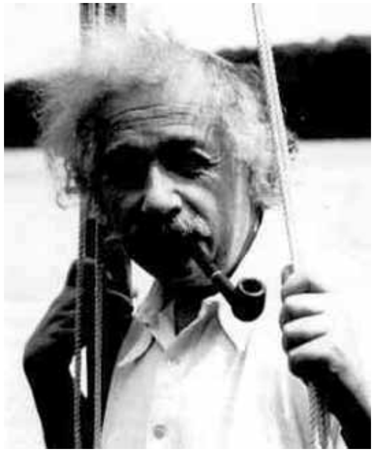
The LXCDM models: the new "cosmon"

Attempting the old CC problem: the "relaxed" universe with a non-fine-tuned $\Lambda$ term

Conclusions

# Cosmological Sum Rule

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}$$



In the FLRW metric,

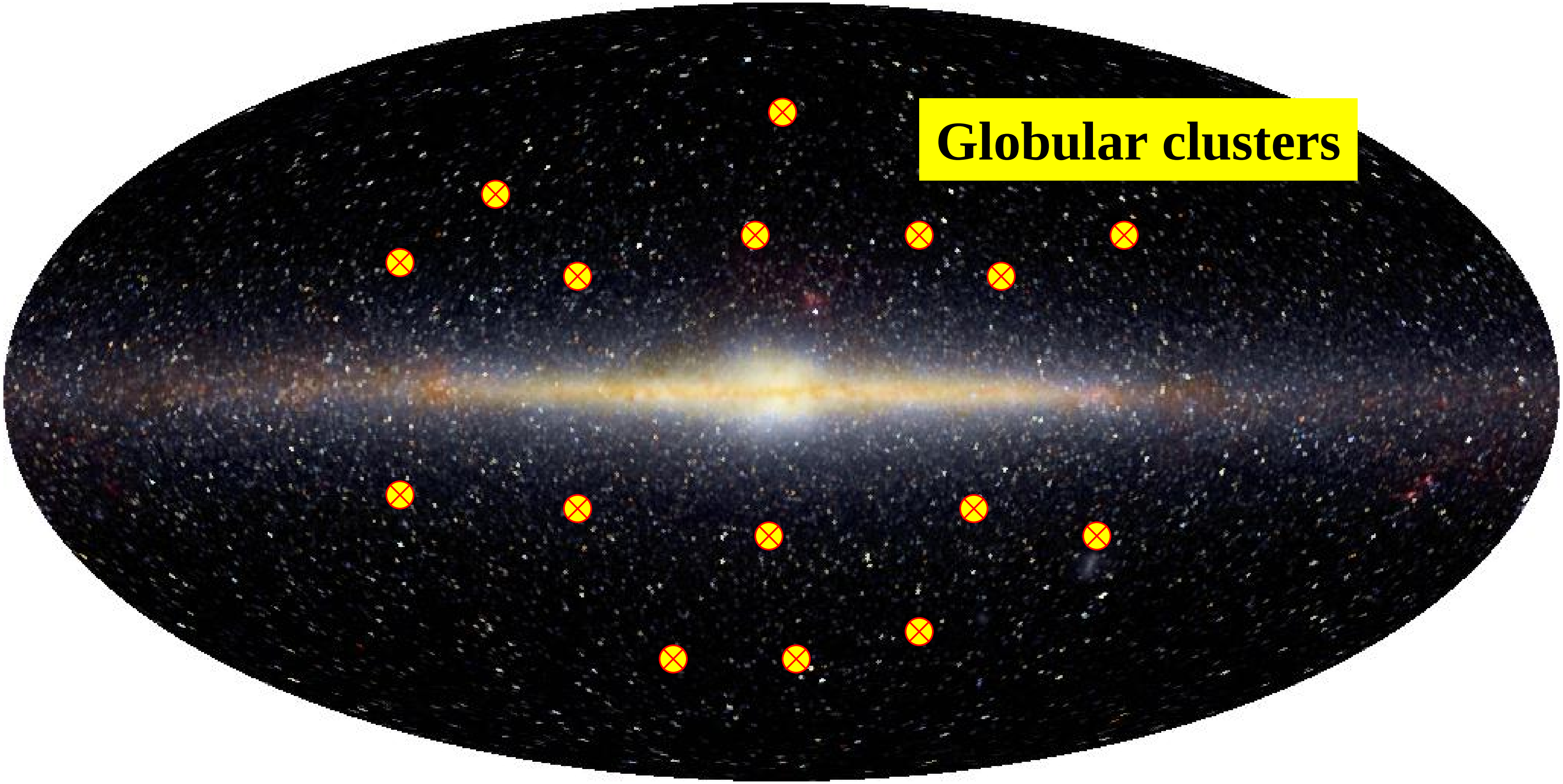
$$\rho_c = \frac{3H^2}{8\pi G_N}$$

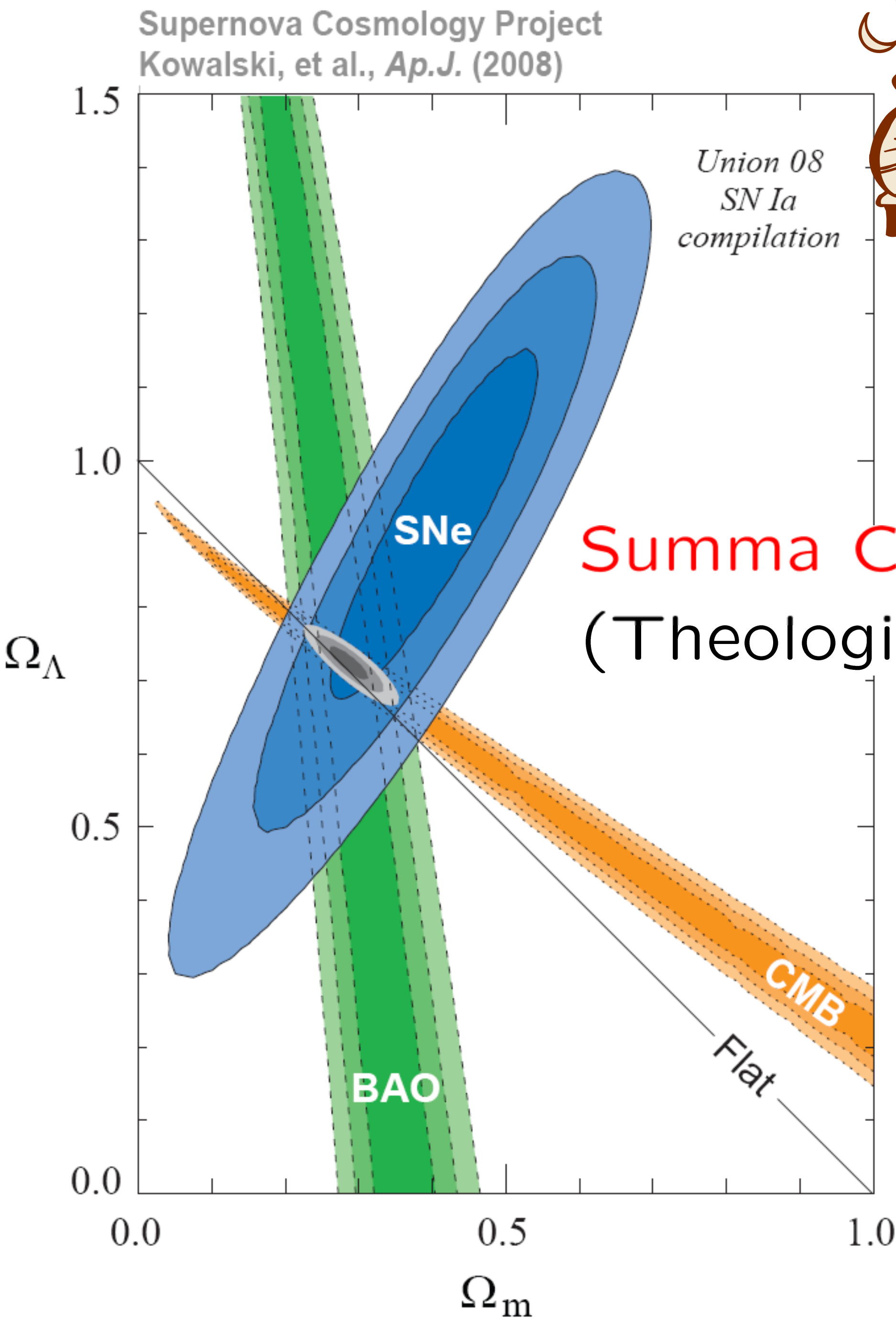
$$\Omega_M = \frac{\rho_M}{\rho_c} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \qquad \Omega_K = -\frac{K}{H^2}$$

$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

# The Milky Way

**How do we know about Λ existence?**



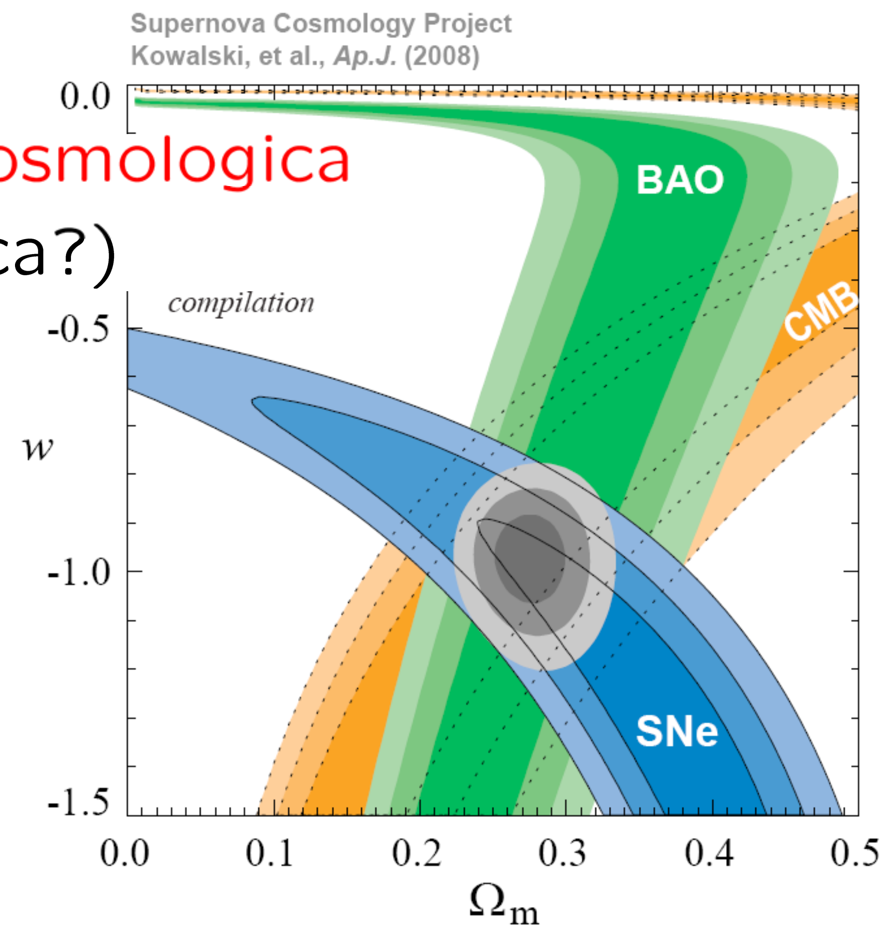



$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

$$\Omega_K \simeq 0$$

# Summa Cosmologica

(Theologica?)



# $\Lambda$ in QFT: the Vacuum Energy

A bit of QFT stuff...

◇ Action integral for a scalar QFT:

$$S[\phi] = \int d^4x \sqrt{-g}\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi - V_{eff}(\phi)$$

◇ Matter field energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 2\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu}\, \mathcal{L}$$

$$= \left[\partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu}\ \partial_\alpha \phi\, \partial^\alpha \phi\right] + g_{\mu\nu} V_{eff}$$

◇ For static equilibrium configurations $\Rightarrow$

$$< T_{\mu\nu} >= g_{\mu\nu} \ < V_{eff} >$$

◇ The Cosmological Constant Problem in modern QFT is the realization that the Physical Cosmological Constant is an Effective Cosmological Constant!! ...a **HUGE** one !!! (Zeldovich 1967; Linde 1974, Veltman 1975):

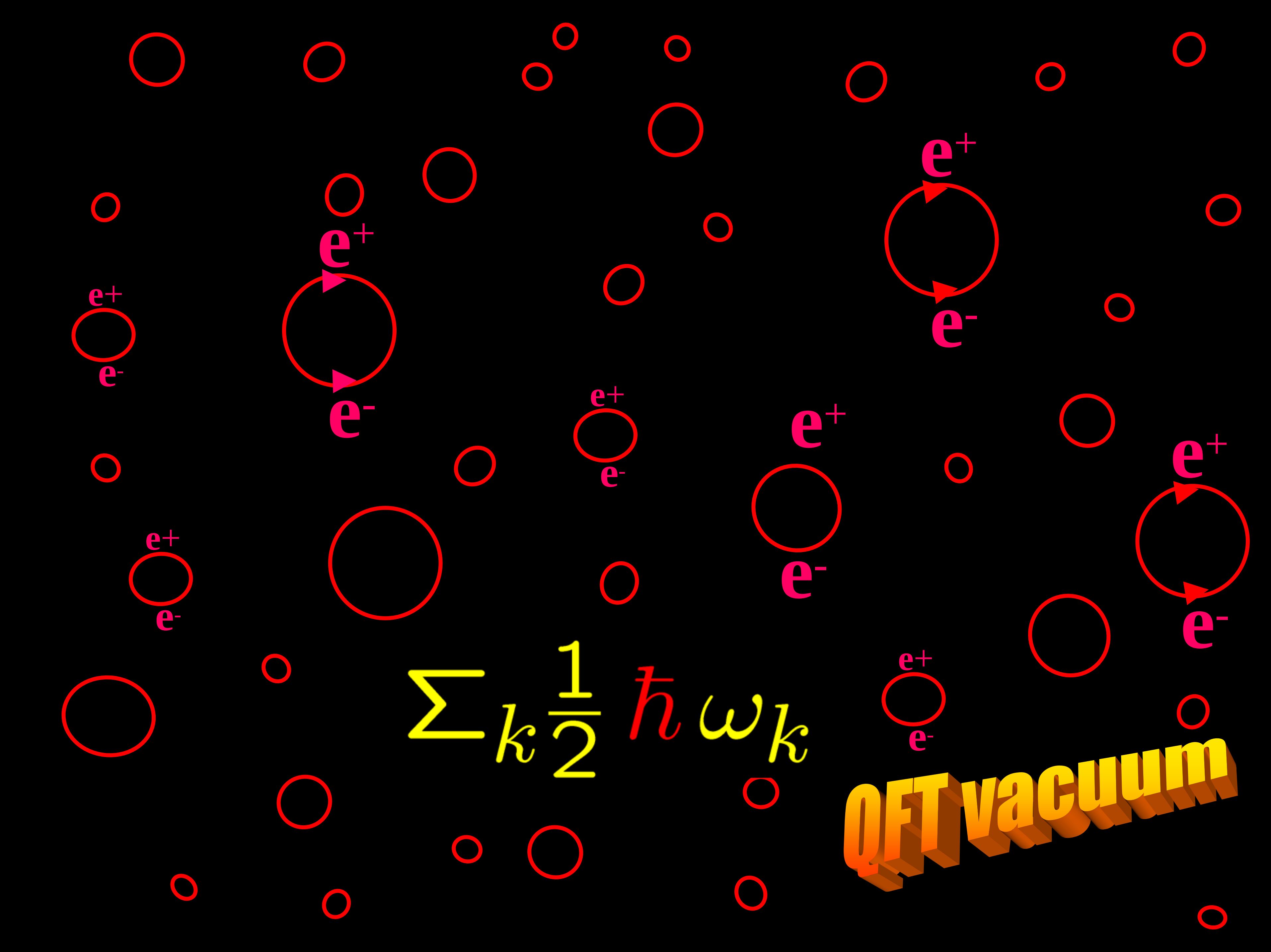
$$\rho_{\Lambda \text{phys}} = \rho_{\Lambda \text{vac}} + \rho_{\Lambda \text{ind}}$$

$$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\, (R - 2\Lambda_{\text{vac}}) = \int d^4x \sqrt{|g|} \left( \frac{1}{16\pi G_N} R - \rho_{\Lambda \text{vac}} \right)$$

Vacuum bare term in Einstein eqs.

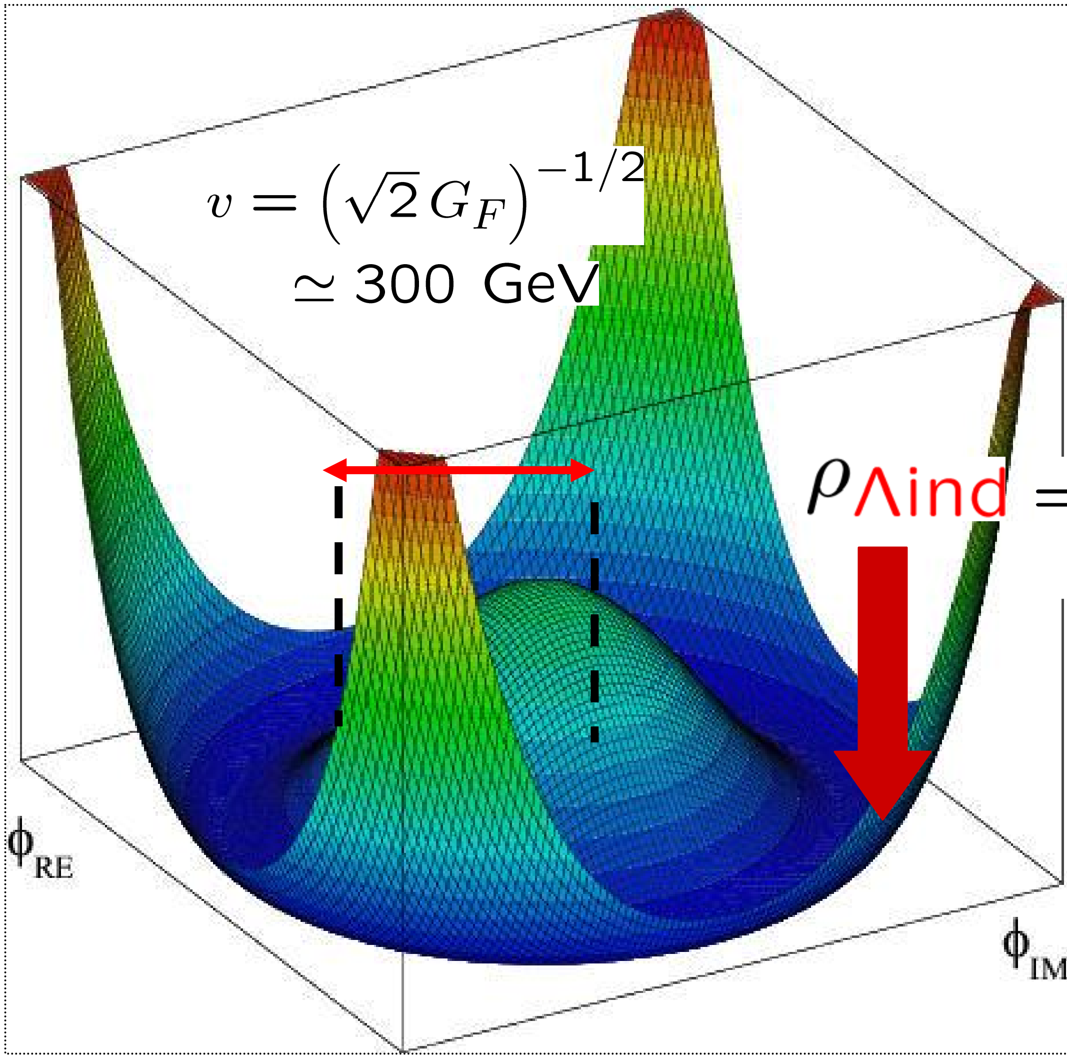
$$R_{ab} - \frac{1}{2} g_{ab} R = -8\pi G_N \left( \langle \tilde{T}^{\varphi}_{ab} \rangle + T_{ab} \right) = -8\pi G_N\, g_{ab} (\rho_{\Lambda \text{vac}} + \rho_{\Lambda \text{ind}} + T_{ab})$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda \text{ind}} = \langle V(\varphi) \rangle +$ ZPE



$e^+$
$e^-$
$e^+$
$e^-$
$e^+$
$e^-$
$e^+$
$e^-$
$e^+$
$e^-$
$e^+$
$e^-$
$e^+$
$e^-$
$e^+$
$e^-$
$\sum_k \frac{1}{2}\hbar\omega_k$
QFT vacuum

# Higgs Potential ⟶ Vacuum Energy




$$V(\varphi) = \frac{1}{2}\, m^2\, \varphi^2 + \frac{1}{4!}\,\lambda\,\varphi^4$$

$$m^2 < 0 \;\;\Rightarrow$$

$$v \equiv \langle\varphi\rangle = \sqrt{\frac{-6\, m^2}{\lambda}}$$

$$\rho_{\Lambda ind} = \langle V(\varphi)\rangle = -\tfrac{1}{8}\, M_{\mathcal{H}}^2\, v^2$$

$$\sim -10^8\ GeV^4 \ \textbf{!!}$$

$$M_W = \tfrac{1}{2} g v$$

$$M_Z = \tfrac{1}{2} v \sqrt{g^2 + g'^2}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

...

$\mu \longrightarrow \nu_\mu \bar{\nu}_e\, e \quad \Rightarrow G_F \quad M_{\mathcal{H}} > 114$ GeV

## Theory:

$$\rho_{\Lambda \mathrm{ind}} = \langle V(\varphi) \rangle = -\frac{1}{8} M_{\mathcal{H}}^2 v^2 \sim -10^8\ GeV^4$$

versus observation:

$$\Omega_\Lambda \simeq 0.7 \quad \Leftrightarrow \quad \rho_{\Lambda \mathrm{ind}} \simeq 10^{-47}\ \mathrm{GeV}^4$$

$$\frac{\rho_{\Lambda \mathrm{ind}}}{\rho_{\Lambda \mathrm{phys}}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55}$$

!!

# ➢ The old CC problem as a fine tuning problem

A little bit more of QFT stuff...

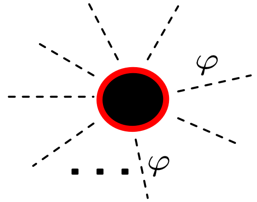
F. Bauer, JS, H. Stefancic
PLB 678 (2009) 427;
PLB 688 (2010) 269
+ arXiv:1006.3944

Take a scalar QFT with effective potential

$$V_{\rm eff} = V + \hbar\, V_1 + \hbar^2\, V_2 + \hbar^2\, V_3 + \ldots$$

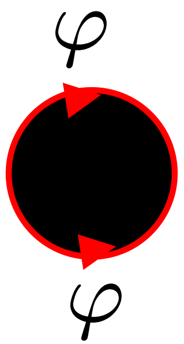
where

$$V_1 = V_P^{(1)} + V_{\rm scal}^{(1)}(\varphi)\,, \qquad V_2 = V_P^{(2)} + V_{\rm scal}^{(2)}(\varphi)\,, \qquad V_3 = V_P^{(3)} + V_{\rm scal}^{(3)}(\varphi)\ldots.$$




Thus,

$$\boxed{V_{\rm eff}(\varphi) = V_{\rm ZPE} + V_{\rm scal}(\varphi)}$$

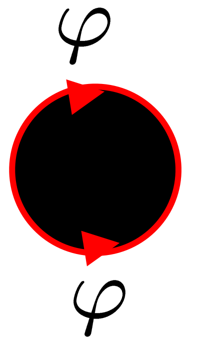



with

$$V_{\rm ZPE} = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots.$$

ZPE at one-loop:

$$\Sigma_k \frac{1}{2} \hbar \, \omega_k$$




$$V_P^{(1)} = \frac{1}{2} \mu^{4-n} \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \sqrt{\vec{k}^2 + m^2}$$

$$= \frac{1}{2} \beta_\Lambda^{(1)} \left( -\frac{2}{4-n} - \ln \frac{4\pi\mu^2}{m^2} + \gamma_E - \frac{3}{2} \right)$$

$$\boxed{\beta_\Lambda^{(1)} = \frac{m^4}{32\,\pi^2}}$$

In the $\overline{\text{MS}}$ scheme,

$$\delta\rho_{\Lambda\text{vac}} = \frac{m^4 \, \hbar}{4\,(4\pi)^2} \left( \frac{2}{4-n} + \ln 4\pi - \gamma_E \right)$$

Renormalized vacuum energy:

$$\rho_\Lambda = \rho_{\Lambda \mathrm{vac}\,0} + V_{\mathrm{ZPE}\,0} = \rho_{\Lambda \mathrm{vac}}(\mu) + V_{\mathrm{ZPE}}(\mu)$$

Renormalized ZPE:

$$\rho_{\Lambda \mathrm{vac}\,0} = \rho_{\Lambda \mathrm{vac}} + \delta\rho_{\Lambda \mathrm{vac}}$$

$$V_{\mathrm{ZPE}}(\mu) = \hbar\, V_P^{(1)} + \delta\rho_{\Lambda \mathrm{vac}}$$

$$\rho_\Lambda = \rho_{\Lambda \mathrm{vac}}(\mu) + \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

Vacuum energy $\rho_\Lambda$ is well-defined and $\mu$-independent

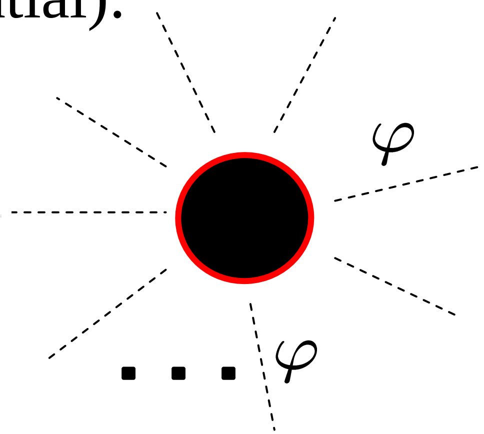
$$\mu \frac{d}{d\mu}\left[\rho_{\Lambda\mathrm{vac}}(m(\mu),\lambda(\mu);\mu)) + V_{\mathrm{ZPE}}(m(\mu),\lambda(\mu);\mu)\right] = 0$$

$$\frac{d\rho_{\Lambda\mathrm{vac}}}{d\ln\mu} = \frac{m^4\,\hbar}{2\,(4\pi)^2} = \beta_\Lambda^{(1)}\,\hbar$$

RGE for $\rho_\Lambda$

In addition, the tail-dependent part (Higgs potential):

$$V_{\rm scal}(\varphi) = V(\varphi) + \hbar\, V^{(1)}_{\rm scal}(\varphi) + \hbar^2\, V^{(2)}_{\rm scal}(\varphi) + $$




At one-loop:

$$m_0 = m + \delta m,\ \lambda_0 = \lambda + \delta\lambda,\ \varphi_0 = Z_{\varphi}^{1/2}\,\varphi = (1 + \delta Z_{\varphi}/2)\,\varphi ...$$

$$V_{\rm eff}(\varphi) = \frac{1}{2}\,m^2(\mu)\,\varphi^2 + \frac{1}{4!}\,\lambda(\mu)\,\varphi^4 + \frac{\hbar\,\left(V''(\varphi)\right)^2}{4(4\pi)^2}\left(\ln\frac{V''(\varphi)}{\mu^2} - \frac{3}{2}\right)$$

$$V''(\varphi) = m^2 + \frac{1}{2}\,\lambda\,\varphi^2$$

$$V_{\rm eff}(\varphi = 0) = \frac{\hbar\,m^4}{4(4\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

Putting everything together:

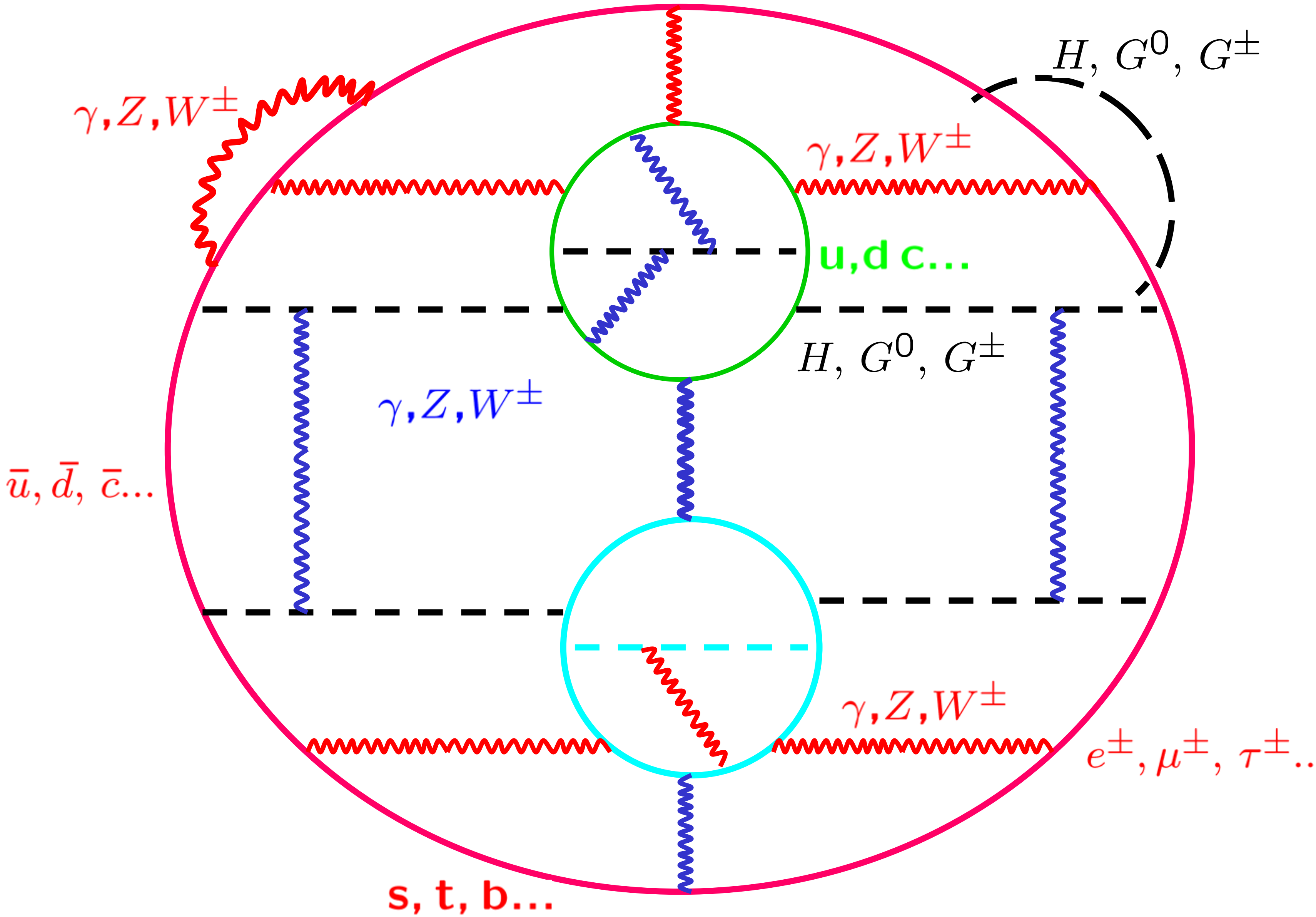
$$\rho_{\Lambda \mathrm{ph}} = \rho_{\Lambda \mathrm{vac}} + \rho_{\Lambda \mathrm{ind}} = \rho_{\Lambda \mathrm{vac}}^{\mathrm{ren}} + \langle V_{\mathrm{eff}}^{\mathrm{ren}}(\varphi)\rangle$$
$$= \rho_{\Lambda \mathrm{vac}}^{\mathrm{ren}} + V_{\mathrm{ZPE}}^{\mathrm{ren}} + \langle V_{\mathrm{scal}}^{\mathrm{ren}}(\varphi)\rangle$$

$$10^{-47}\,\mathrm{GeV}^4 = \rho_{\Lambda \mathrm{vac}} - 10^8\,\mathrm{GeV}^4 + \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} \ldots.$$
$$+ \hbar\, V_{\mathrm{scal}}^{(1)}(\varphi) + \hbar^2\, V_{\mathrm{scal}}^{(2)}(\varphi) + \hbar^3 V_{\mathrm{scal}}^{(3)}(\varphi)\ldots$$

With $v \sim 100$ GeV, which is the highest loop involved?:

$$\left(\frac{g^2}{16\,\pi^2}\right)^n v^4 = 10^{-47}\,GeV^4 \qquad \Rightarrow \boxed{n \simeq 21}\ \mathbf{!!}$$

21th loop (one among many thousands...)



# The many Cosmological Constant Problems

S. Weinberg, Rev.Mod.Phys.61 (1989) 1

In the SM, $\Lambda_{\rm ph} = \Lambda_v + \Lambda_{SM}$

- **Problem I**:

The "Classic" CC Problem:

$$\left(\frac{\Lambda_{SM}}{\Lambda_{\rm ph}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55}\right)$$

**Why the induced and vacuum counterparts of the CC cancel each other with such a huge precision?**

F. Bauer, JS, H. Stefancic
PLB 678 (2009) 427;
PLB 688 (2010) 269
+ arXiv:1006.3944

- **Problem II**:

The (first) "Coincidence" CC Problem:

J.Grande, A. Pelinson, JS,
PRD 79 (2009) 043006
JCAP 0712:007,2007.
JCAP 0608:011,2006.

**Why the observed CC in the present-day Universe is so close to the matter density $\rho$?**

coincidence ratio now:

$$r \equiv \frac{\rho_\Lambda^0}{\rho_M^0} = \frac{\Omega_\Lambda^0}{\Omega_M^0} \simeq \frac{7}{3} = \mathcal{O}(1)$$

- **Problem III**:

The "nature" of the the CC Problem:

In more recent times the notion of $\Lambda$ has been superseded by that of the DE. The latter is more general and involves a variety of models leading to an accelerated expansion of the universe in which the DE itself is a time-evolving entity. These models include dynamical scalar fields (quintessence... and the like), phantom fields, braneworld models, Chaplygin gas, holographic dark energy, cosmic strings, domain walls...

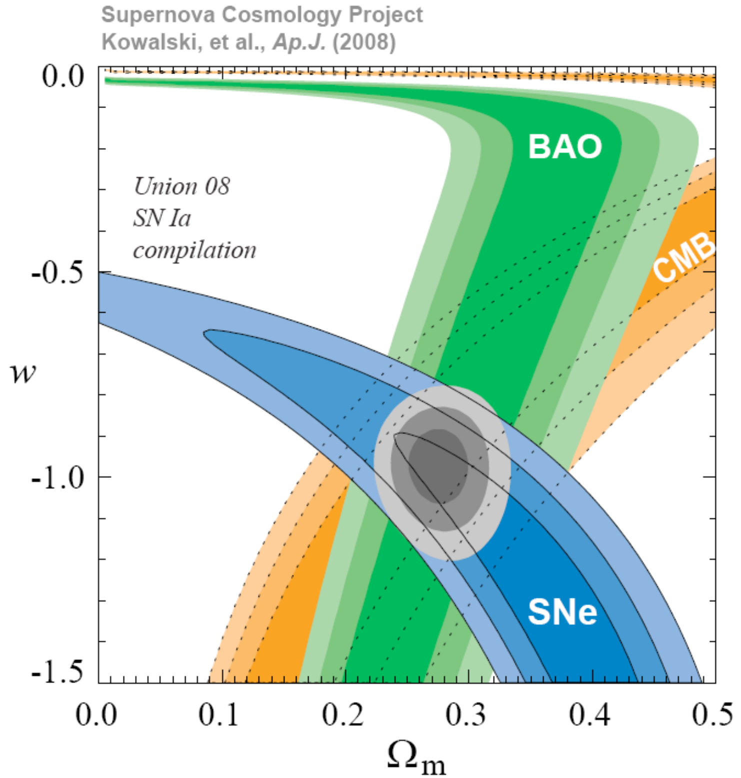
What is, then, the true dynamical cause responsible for the DE?

- Problem IV:

The (second) "Coincidence" Problem:

Present observations seem to indicate an evolving DE with a potential phantom phase near our time.

If the dark energy behaves phantom-like, why just now?



# ➢ ''Canonical'' definition of Dynamical Dark Energy

One popular possibility is the idea of quintessence, where there is no "true" $\Lambda$

The total energy-momentum tensor on the *r.h.s.* of Einstein eqs. is the sum

$$\tilde{T}_{\mu\nu} \equiv T^{M}_{\mu\nu} + T^{D}_{\mu\nu}\,.$$

One assumes that both tensors are separately conserved, and so $\bigtriangledown^{\mu}\tilde{T}_{\mu\nu} = 0$ is equivalent to

$$\bigtriangledown^{\mu} T^{M}_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

and (unmixed conservation laws)

$$\bigtriangledown^{\mu} T^{D}_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho_D}{dt} + 3\,H\,(\rho_D + p_D) = 0$$

**Nice feature of quintessence field:**

$$\omega_D = \frac{p_D}{\rho_D} = \frac{\frac{1}{2}\xi\dot{\chi}^2 - V(\chi)}{\frac{1}{2}\xi\dot{\chi}^2 + V(\chi)} \simeq -1 + \xi\,\dot{\chi}^2/V(\chi)$$

**Problems with quintessence field: forgets SM vacuum!!**

Even taking the simplest form $V(\chi) = (1/2)\, m_\chi^2\, \chi^2$

$$\Downarrow \quad \rho_\Lambda^0 = V(\chi)$$

$$\chi \simeq M_X \simeq 10^{16}\, GeV \quad \Rightarrow \quad m_\chi \simeq H_0 \simeq 10^{-33}\, eV$$

$$\chi \simeq M_F = G_F^{-1/2} \quad \Rightarrow \quad m_\chi \simeq 10^{-12}\, eV$$

(Recall that $m_\Lambda \sim$ meV $\Rightarrow$ billion times)**!**

Question:

Can a dynamical DE still be $\Lambda$?...

# Need $\Lambda = \Lambda(t)$ !!

But still $w = -1$...!! $\quad T_{\mu\nu} \Rightarrow \tilde{T}_{\mu\nu}$

$$\tilde{T}_{\mu\nu} = \rho_\Lambda(t) - p\, g_{\mu\nu} + (\rho + p)\, U_\mu U_\nu \quad = -\tilde{p}\, g_{\mu\nu} + (\tilde{\rho} + \tilde{p})\, U_\mu U_\nu$$

$$\tilde{p} = p - \rho_\Lambda(t) \qquad \tilde{\rho} = \rho + \rho_\Lambda(t) \qquad \boxed{\Rightarrow p_\Lambda(t) = -\rho_\Lambda(t)}$$

DE picture of it: $\dot{\rho_D} + 3\, H_D(1 + \omega_D)\rho_D = 0$

# Generic time-varying CC models versus observation

S. Basilakos, M. Plionis, JS, Phys.Rev.D80 (2009)
(see talk by S. Basilakos)

1) Quantum field vacuum ($\Lambda_{RG}$)

$$\Lambda(H) = n_0 + n_2 H^2$$

I.L.Shapiro, JS.
*JHEP 0202 (2002) 6*
Phys.Lett.B475 (2000) 236.

2) Power series model ($\Lambda_{PS1}$)

$$\Lambda(H) = n_1 H + n_2 H^2$$

S. Basilakos
MNRAS 395 (2009) 2347

3) Linear model ($\Lambda_{PS2}$)

$$\Lambda(H) \propto H$$

R. Schutzhold, PRL 89 (2002)
S. Carneiro et al. (2008), F. Klinkhammer and G.E. Volovik (2009) etc

4) Quadratic model

$$\Lambda(H) \propto H^2 \propto \rho_T$$

J.C. Carvalho et al (1992),
R.C. Arcuri and I. Waga (1994) etc.

5) Power law model ($\Lambda_n$ )

$$\Lambda(H) \propto a^{-n}$$

M. Ozer and O. Taha (1987),
W. Chen and Y.S. Wu (1990)

# Variable $\Lambda$

- For variable $\Lambda$, the conserved quantity is not the matter energy-momentum tensor $T_{\mu\nu}$, but the sum

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + g_{\mu\nu}\, \rho_\Lambda(t)\,, \qquad \nabla^\mu \tilde{T}_{\mu\nu} = 0\,.$$

By the Bianchi identities, $\Lambda$ is constant $\Longleftrightarrow$ the matter $T_{\mu\nu}$ is individually conserved ($\nabla^\mu T_{\mu\nu} = 0$)– in particular, $\rho_\Lambda =$ const. if $T_{\mu\nu} = 0$ (e.g. during inflation).

- From FLRW metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - k\, r^2} + r^2\, d\theta^2 + r^2\, \sin^2\theta\, d\phi^2\right)\,,$$

we may compute explicitly the local energy-conservation law $\nabla^\mu \tilde{T}_{\mu\nu} = 0$. The result is an equation allowing transfer of energy between ordinary matter and the dark energy associated to the $\Lambda$ term :

$$\frac{d\rho_\Lambda}{dt} + \frac{d\rho}{dt} + 3\, H\, (\rho + p) = 0,$$

(<u>mixed</u> conservation law!)

## A semiclassical FLRW with running $\Lambda$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) + H_0^2\,\Omega_K^0(1+z)^2$$

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0\,.$$

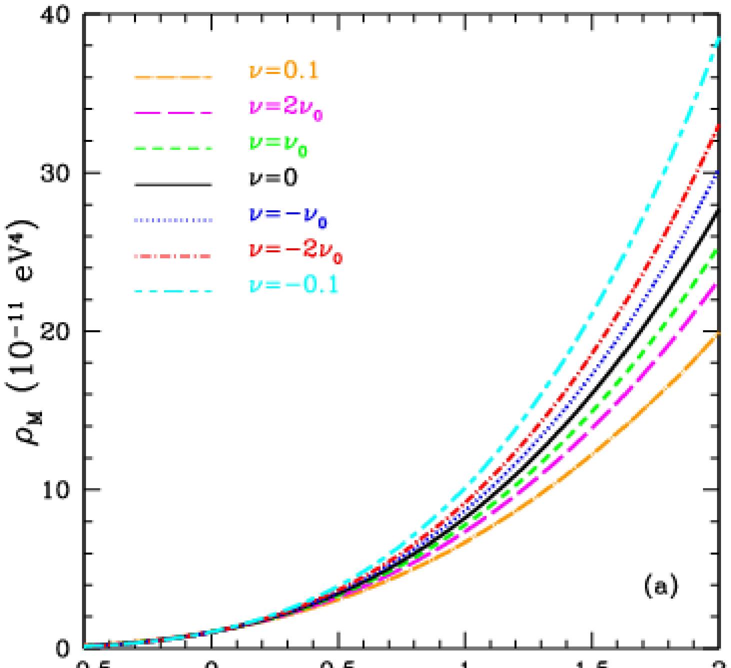
$$\frac{d\Lambda}{d\ln H} = \frac{1}{(4\pi)^2}\sum_i c_i\, M_i^2\, H^2 + \ldots = \frac{3\nu}{4\pi} M_P{}^2 H^2.$$

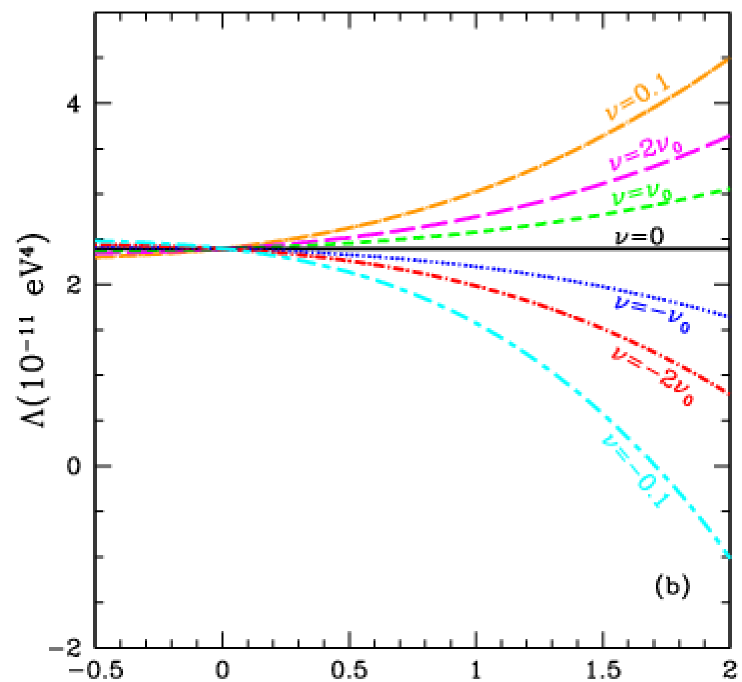
$$\nu = \frac{\sigma\,\mathrm{M}^2}{12\pi M_P{}^2}$$

$$\rho_\Lambda \equiv \Lambda = C_1 + C_2\, H^2\,.$$

Effects on $\rho_{\mathrm{M}}$ and $\Lambda$, for $\Omega_M^0 = 0.3$, $\Omega_\Lambda^0 = 0.7$

$$\rho_m(z) = \rho_m^0\,(1+z)^{3(1-\nu)} \qquad \rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu\,\rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu))} - 1\right]$$




$$\nu = \frac{\sigma\,\mathrm{M}^2}{12\pi\,{M_P}^2}$$

$$\nu_0 \equiv \frac{1}{12\,\pi} \simeq 2.6 \times 10^{-2}$$

C. España-Bonet et al., *JCAP* **0402** (2004) 6.

S. Basilakos, M. Plionis, JS,
Phys.Rev.D80 (2009) ⇒ Basilakos talk

## Running both... $G$ and $\Lambda$ ?

I.L. Shapiro, J.S., H. Stefancic
*JCAP* 0501 (2005)

J. S., J.Phys.A41 (2008)

**Bianchi identity leads to** $\nabla^{\mu}\left[G\left(T_{\mu\nu}+g_{\mu\nu}\,\rho_{\Lambda}\right)\right]=0$

$$\frac{d}{dt}\left[G(\rho_m+\rho_{\Lambda})\right]+3\,G\,H\,(\rho_m+p_m)=0\,.$$

**Possible scenario:**

$$\dot{G}\neq 0 \;\; and \;\; \dot{\rho_{\Lambda}}\neq 0 \Rightarrow \qquad \dot{\rho}_m+3\,H(\rho_m+p_m)=0$$

$$(\rho+\rho_{\Lambda})\dot{G}+G\dot{\rho_{\Lambda}}=0$$

# Running G logarithmically…

**Basic set of equations:**
**(k=0)**

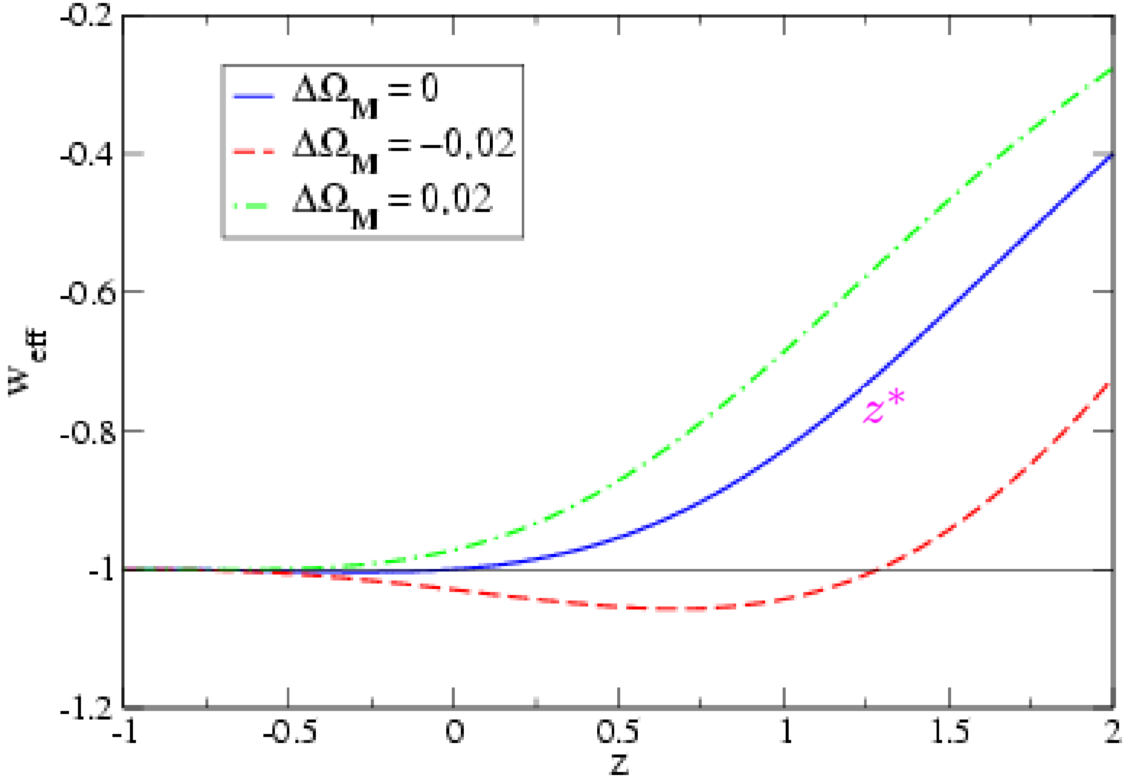
$$\begin{cases} \rho + \rho_\Lambda = \dfrac{3H^2}{8\pi G}\,, \\ \rho_\Lambda = C_1 + C_2 H^2\,, \\ (\rho + \rho_\Lambda)\, dG + G\, d\rho_\Lambda = 0 \end{cases}$$

$$C_1 = \rho_\Lambda^0 - \frac{3\nu}{8\pi} M_P^2 H_0^2\,, \qquad C_2 = \frac{3\nu}{8\pi} M_P^2$$

$$G(H;\nu) = \frac{G_0}{1 + \nu \ln\left(H^2/H_0^2\right)}$$

$H \to a$ in J. S., J.Phys.A41 (2008)

# Crossing the phantom divide...




J.Solà, H. Stefancic
*Phys. Lett. B 624 (2005) 147;*
*Mod. Phys. Lett. A21 (2006)* 479

Using phantom scalar fields:
R.R. Caldwell, PLB 545 (2002) 23
A. Melchiorri et al.
PRD 68 (2003) 043509
B. Feng, X.L. Wang, X.M. Zhang,
PLB 607 (2005) 35

$$\nu = \frac{1}{12\pi}\frac{M^2}{M_P^2} \qquad (\text{case } \nu = -\nu_0 < 0)$$

## ➢ ΛXCDM model without fine-tuning

F. Bauer, JS, H. Stefancic
PLB 678 (2009) 427;
PLB 688 (2010) 269
+ arXiv:1006.3944

The "Relaxed Universe"

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[ \frac{1}{16\pi G_N} R - \boxed{\rho_\Lambda^i} - \mathcal{F}(R,\mathcal{G}) + \mathcal{L}_\varphi \right]$$

Arbitrarily large

Field equations:

$$G^a_{\ b} = -8\pi G_N \left[ \boxed{\rho_\Lambda^i} \delta^a_{\ b} + 2E^a_{\ b} + T^a_{\ b} \right]$$

$$E^0_{\ 0} = \frac{1}{2} \left[ \mathcal{F}(R,\mathcal{G}) - 6(\dot{H} + H^2)\mathcal{F}^R + 6H\dot{\mathcal{F}}^R - 24H^2(\dot{H} + H^2)\mathcal{F}^\mathcal{G} + 24H^3\dot{\mathcal{F}}^\mathcal{G} \right]$$

$$E^i_{\ j} = \frac{1}{2}\delta^i_{\ j} \left[ \mathcal{F}(R,\mathcal{G}) - 2(\dot{H} + 3H^2)\mathcal{F}^R + 4H\dot{\mathcal{F}}^R + 2\ddot{\mathcal{F}}^R - 24H^2(\dot{H} + H^2)\mathcal{F}^\mathcal{G} + 16H(\dot{H} + H^2)\dot{\mathcal{F}}^\mathcal{G} + 8H^2\ddot{\mathcal{F}}^\mathcal{G} \right],$$

where

$$\begin{cases} R = 6H^2(1-q) & \text{(FLRW metric)} \\ \mathcal{G} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} = -24H^4 q \\ q = -1 - \frac{\dot{H}}{H^2} \end{cases}$$

The effective CC term is of ΛXCDM type:

$$\rho_{\Lambda\text{eff}}(H) = \rho_\Lambda^i + \rho_{\text{ind}}(H)$$

$$\rho_{\text{ind}} = 2E_0^0 \equiv \rho_F \qquad \text{(induced DE )}$$

To counterbalance $\rho_{\Lambda}^{i}$ dynamically $\Rightarrow$

$$\boxed{\mathcal{F}(R,\mathcal{G}) = \beta\, F(R,\mathcal{G}) + A(R)}$$

$$F(R,\mathcal{G}) = \frac{1}{B(R,\mathcal{G})}$$

$$\Downarrow$$

$$B(R,\mathcal{G}) = \frac{2}{3}R^2 + \frac{1}{2}\mathcal{G} + (y\,R)^n$$

$$\Downarrow$$

$$= 24H^4\left(q - \tfrac{1}{2}\right)(q-2) + \left[6yH^2(1-q)\right]^n$$

Let us consider a toy model first:

$$B = R^2 = 36\, H^4 (1-q)^2$$

$$\rho_{\Lambda\mathrm{eff}}(H) = \rho_\Lambda^i + \rho_F = \rho_\Lambda^i + \beta\, \left[\frac{1}{36\, H^4 (1-q)^2} + \mathcal{E}(H, q, \dot{q})\right]$$

$$= \rho_\Lambda^i + \beta\, \left[\frac{\mathcal{N}_2}{(1-q)^2} + \frac{\mathcal{N}_3}{(1-q)^3} + \frac{\mathcal{N}_4}{(1-q)^4}\right]$$

Take the SM characterized by a large $\rho_\Lambda^i < 0$

Tends to produce a dramatic deceleration !

But the dynamical mechanism drives $q \rightarrow 1$ fast!!

The large CC becomes compensated and the radiation era is triggered!

A small value of the CC and $H$ is predicted in the late universe:

$$F(R,\mathcal{G}) = \frac{1}{B(R,\mathcal{G})} \sim \frac{1}{H^4(1-q)}$$

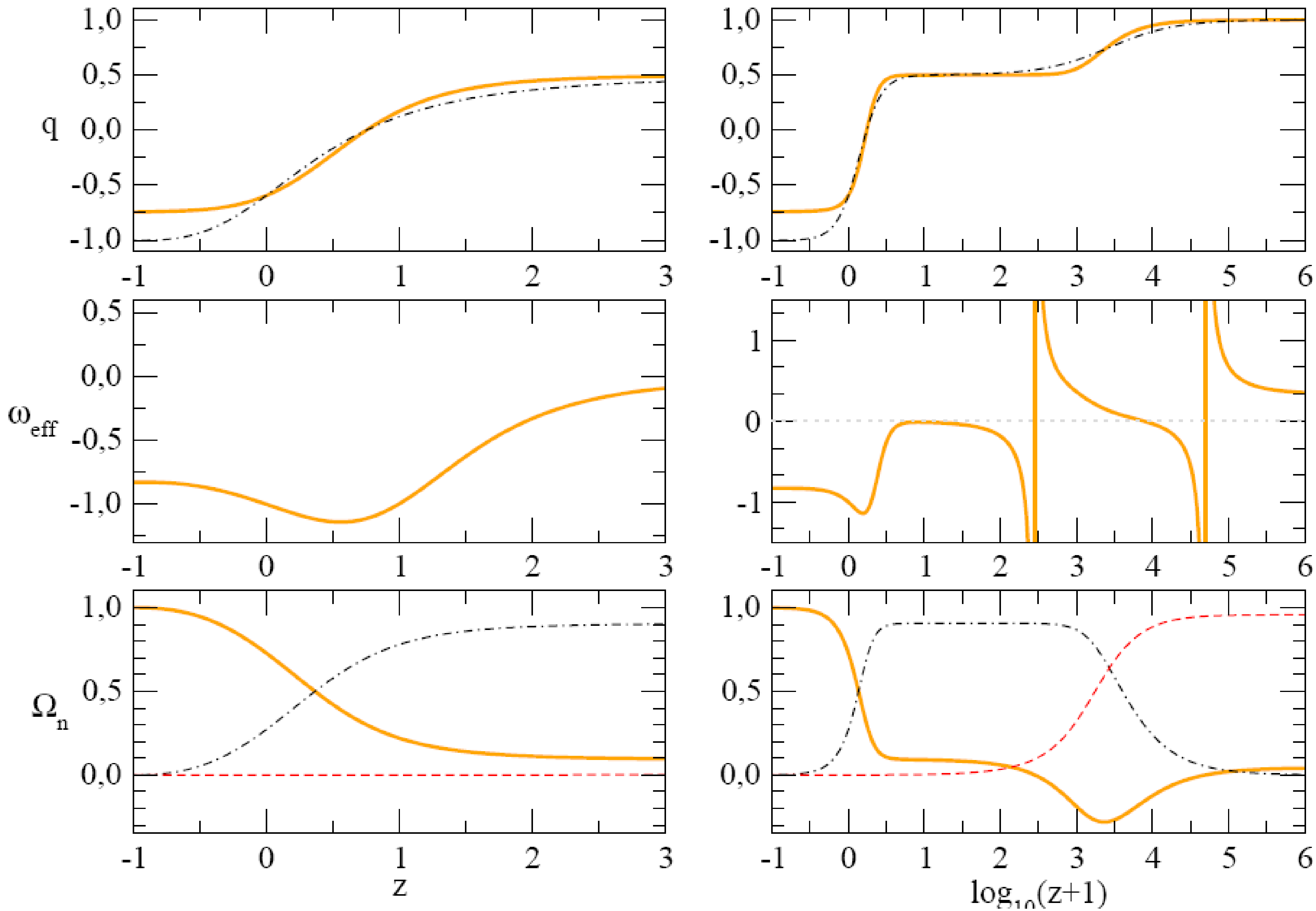
In the present epoch, $q \simeq 1$ ceased to hold, hence the universe must choose a very small value $H \to H_0$ in order to implement the so-called relaxation condition $B \to 0$

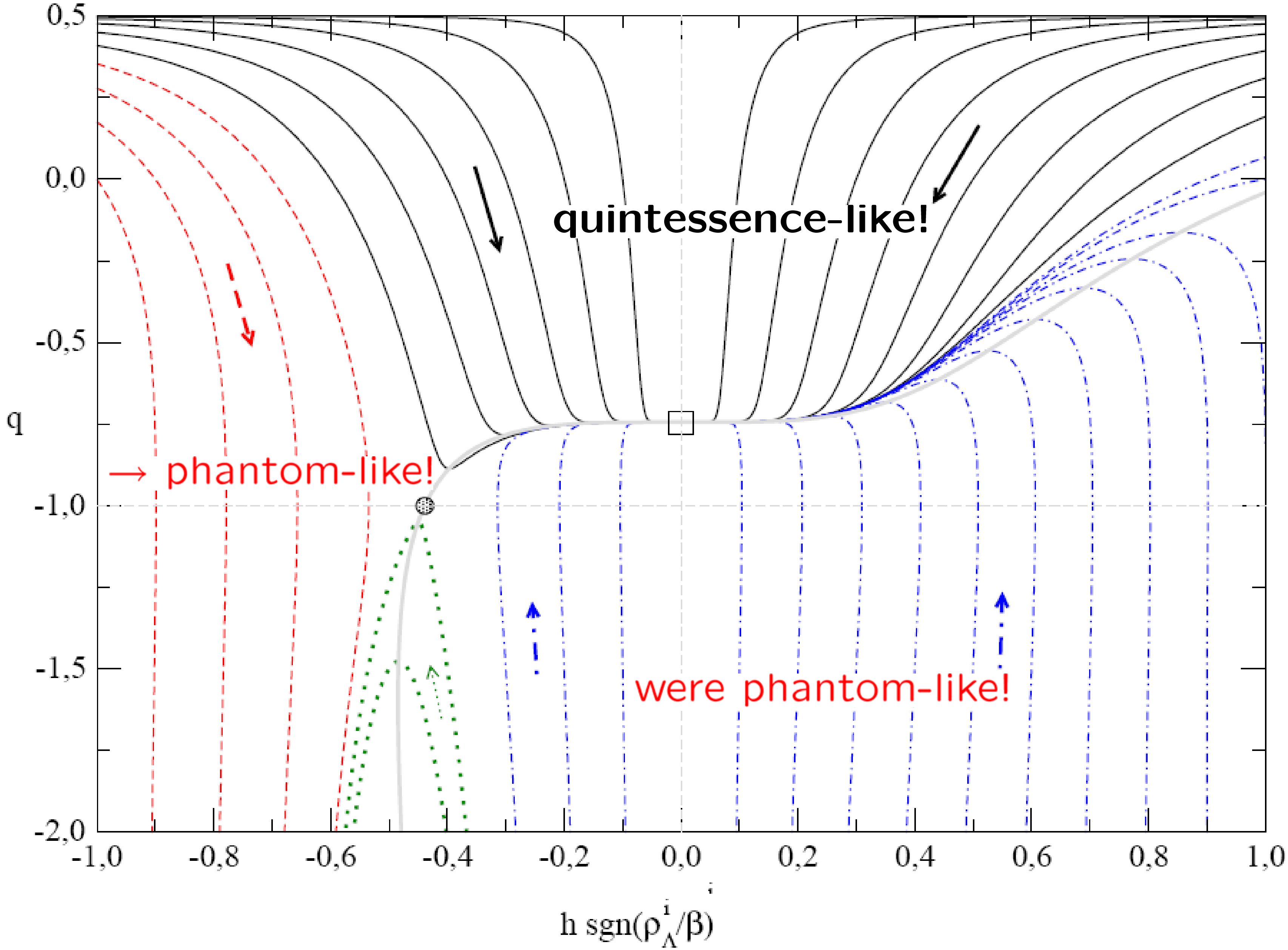
$$\rho_{\Lambda\text{eff}} \simeq \rho_\Lambda^i + \frac{\beta}{H^4(1-q)} \simeq 0$$

$$H_0 \sim \left(\frac{\beta}{|\rho_\Lambda^i|}\right)^{1/4}$$

Small because $\rho_\Lambda^i$ is so large!!

Taking now the full model ⇒ all the eras right!





quintessence-like!
→ phantom-like!
were phantom-like!
q
h sgn(ρ$^i_Λ$/β)
0,5
0,0
-0,5
-1,0
-1,5
-2,0
-1,0
-0,8
-0,6
-0,4
-0,2
0,0
0,2
0,4
0,6
0,8
1,0

## Generalized relaxation models

$$F_m^s := \frac{R^s}{B^m} = \frac{R^s}{\left[\frac{2}{3}R^2 + \frac{1}{2}\mathcal{G} + (y\,R)^n\right]^m}\,, \qquad (s,m>0;\ n>2)$$

$$|\beta| = M^{4-2s+4m}$$

Assume $\rho^i_\Lambda \simeq M_X \simeq 10^{16}$ GeV

- Canonical $F_1^0 \Rightarrow\ M \simeq 10^{-4}$ eV $\sim m_\Lambda \sim m_\nu$
- Next-canonical $F_1^1$ and $F_2^3\ \Rightarrow M \simeq 0.1$ GeV $\sim \Lambda_{\text{QCD}}$
- Next-to-next $F_1^2 \Rightarrow\ M \simeq 10^{16}$ GeV $\sim M_X \simeq \rho^i_\Lambda$

No ultralight mass scales anywhere **!!**

$\Rightarrow$ A large class of $\Lambda$XCDM models can solve the fine-tuning problem and hence could have great impact on the old CC problem !!