Higher Order Corrections to the Drell-Yan Cross Section in the Mellin Space

Petra Kovačíková (DESY, Zeuthen)

petra.kovacikova@desy.de

Corfu Summer School

4^{th} September 2010

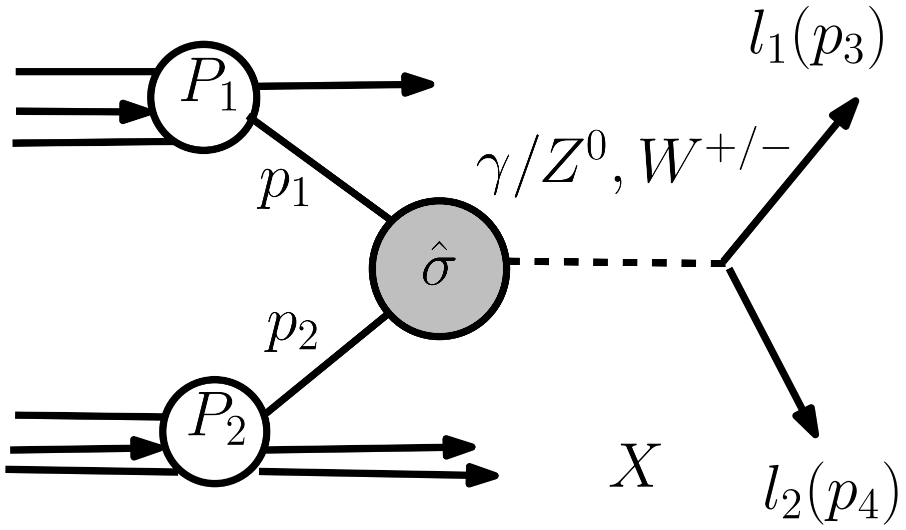
Outline

- Introduction
  - Drell-Yan mechanism
- The Drell-Yan hadronic Cross Section
  - NLO α_s corrections
  - Parton distribution functions
  - Mass fatorization
- The Calculation of Drell-Yan cross section using Mellin transform
- Results and Checks
- Conclusions and Outlook

The Drell-Yan Process

Massive lepton pair production in hadron-hadron collision, $M^2_{l_1 l_2} >> 1$ GeV^2

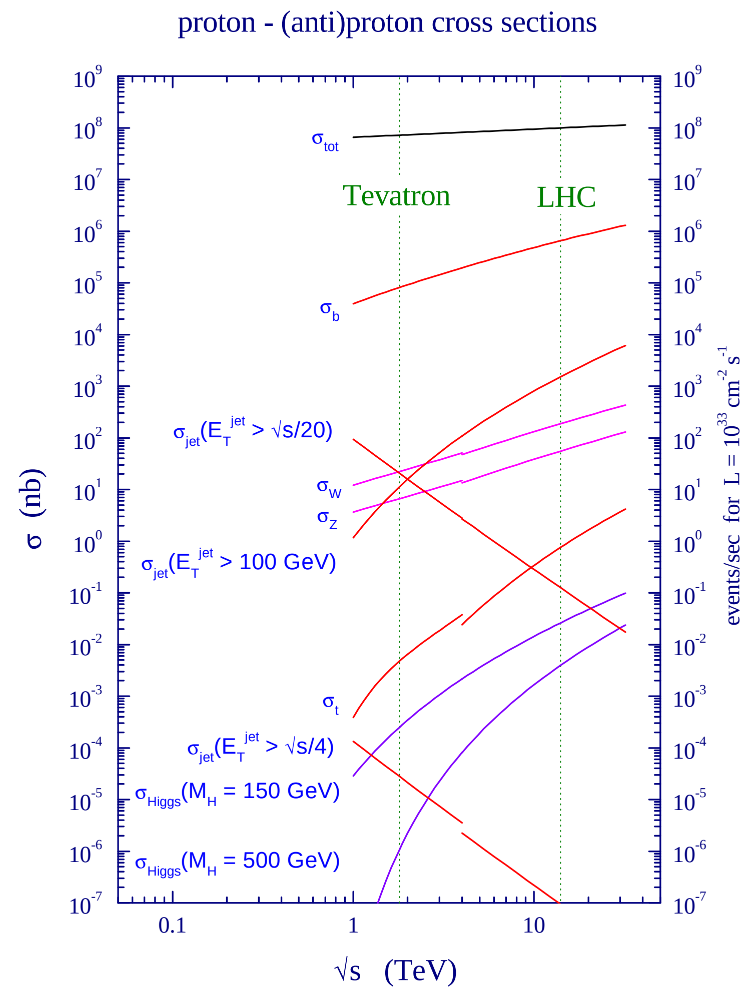
[Drell,Yan 1970]



- $M_{l_1 l_2} = Q^2 = (p_2 + p_3)^2$
- CM energy of hadrons
  $s = (P_1 + P_2)^2$
- $p_i = x_i P_i \qquad x_i \in (0, 1)$
- CM energy of partons
  $\hat{s} = (p_1 + p_2)^2 = s x_1 x_2$

- Neutral Current
  - $pp \to \gamma^* \to l\bar{l}X \qquad M_{l\bar{l}} \ll M_Z \qquad M_Z \sim 91.2$ GeV
  - $pp \to Z^0 \to l\bar{l}X \qquad M_{l\bar{l}} \sim M_Z$
- Charged Current
  - $pp \to W^{\pm} \to l\nu X \qquad M_{l\bar{l}} \sim M_W \qquad M_W \sim 80.4$ GeV

Drell-Yan at the Tevatron and the LHC



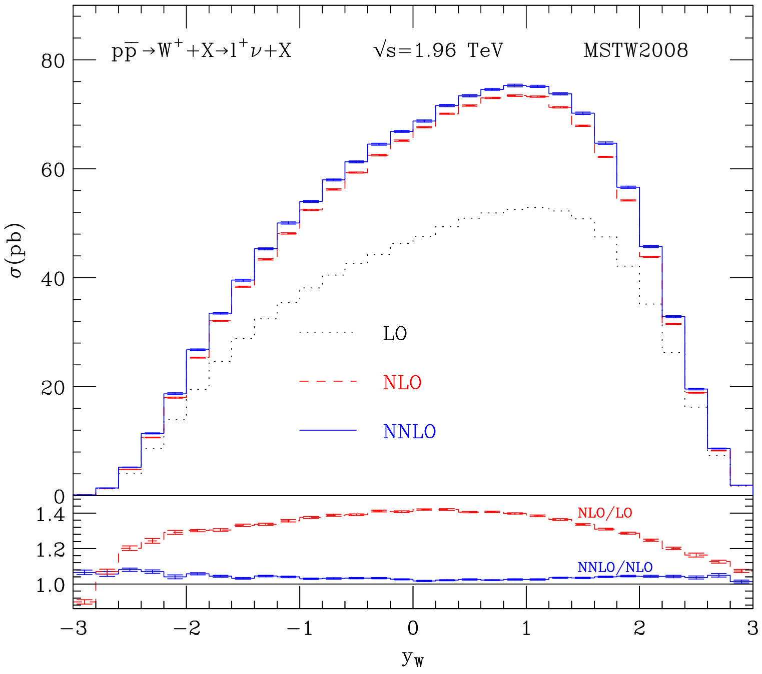
- Large total cross sections for W and Z production
- Number of events $N \sim \int \sigma L \, \mathrm{d}\, t$
- LHC
  W: $\int L \, \mathrm{d}\, t \sim 300\,\text{nb}^{-1}$, $N \sim 2000$
  Z: $\int L \, \mathrm{d}\, t \sim 230\,\text{nb}^{-1}$, $N \sim 150$
  [Gianotti,ATLAS collaboration ICHEP, July 2010]
- Clear signal
- Backrgound for new phyiscs measurements
- Detector calibration, luminosity monitoring, constraints on PDFs

The Drell-Yan Process

- Higher order NLO QCD corrections [Altarelli, Ellis, Martinelli 1979]
- NNLO [Hamberg, Matsuura, van Neerven 1990; Haarlander, Kilgore 2002]
- NNLO double differential cross section [Anastasiou, Dixon, Melnikov, Petriello 2004; Catani, Ferrera, Grazzini 2010]
  $\Rightarrow$ new constraints on PDFs [Alekhin, Melnikov, Petriello 2006]
- Electroweak corrections up to NLO [Hollik, Wackeroth 1996; Vicini et.al. 2009]

The Hadronic Cross Section

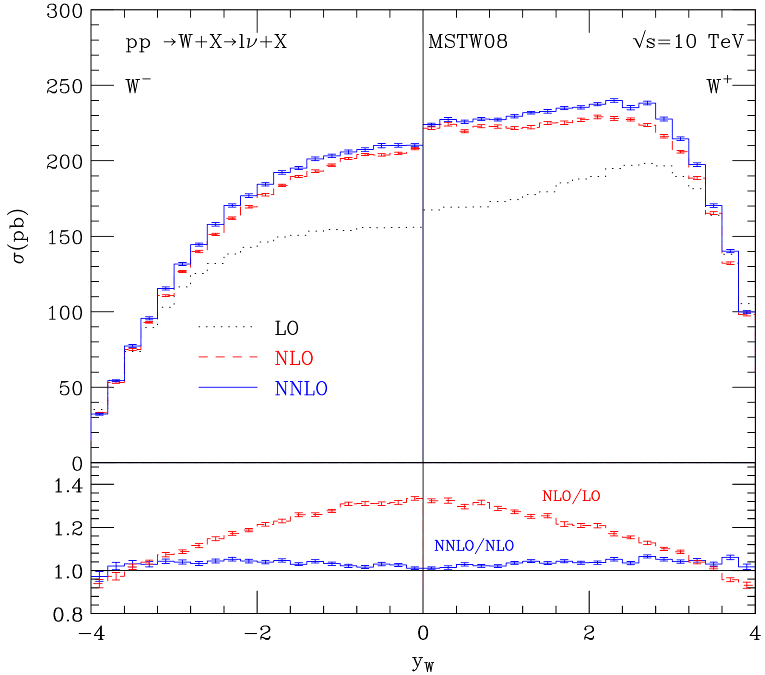
Tevatron



NLO/LO $= 1.3 - 1.4$

NNLO/NLO $= 1.02 - 1.04$

LHC (10 TeV)



NLO/LO $= 1.1 - 1.3$

NNLO/NLO $= 1.01 - 1.05$

- $y_W = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$

[Catani, Ferrera, Grazzini 2010]

The Drell-Yan Process

- Hadronic cross section

$$\frac{\mathrm{d}\,\sigma^V_{pp\to l_1 l_2}}{\mathrm{d}\,Q^2} = \sum_{a,b}\int_0^1 \mathrm{d}\,x_1 \int_0^1 \mathrm{d}\,x_2 \underbrace{f_a(x_1)f_b(x_2)}_{\text{PDFs}} \underbrace{\frac{\mathrm{d}\,\hat{\sigma}^V_{ab}(x_1,x_2,Q^2,\alpha_s(\mu_r^2))}{\mathrm{d}\,Q^2}}_{\text{Hard Cross section}}$$

$$V=\gamma^*, Z^0, W^\pm \qquad a,b=q,\bar{q},g \qquad x_{1,2}\in(0,1)$$

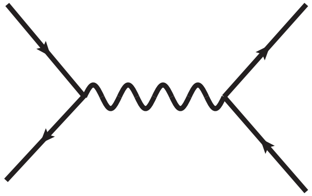
- Partonic cross section up to NNLO

$$\frac{\mathrm{d}\,\hat{\sigma}^V_{ab\to l\bar{l}}}{\mathrm{d}\,Q^2} = \underbrace{\mathcal{N}^V}_{\text{Norm. factor}} \times \underbrace{\mathcal{C}^V}_{\text{EW couplings}} \times \underbrace{\Delta_{ab}(x)}_{\text{Coefficient functions}}$$

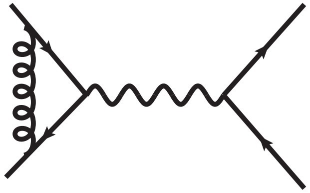
$$\Delta_{ab}(x) = \Delta^{(0)}_{ab}(x) + \frac{\alpha_s}{4\pi}\Delta^{(1)}_{ab}(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 \Delta^{(2)}_{ab}(x), \qquad x=\frac{Q^2}{\hat{s}}$$

The Partonic Cross Section

- Leading order $\Delta_{qq}^{(0)}(x) = \delta(1-x)$

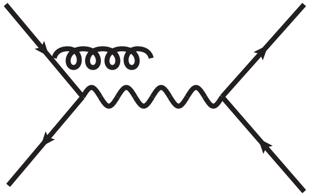


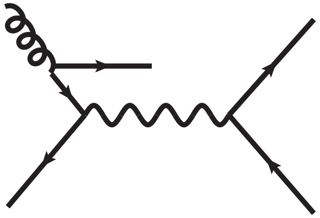
- Next-to-leading order $\hat{\sigma}_n^{\text{NLO}} = \int_n \mathrm{d}\,\hat{\sigma}_{\text{virtual}} + \int_{n+1} \mathrm{d}\,\hat{\sigma}_{\text{real}}$
  - Virtual contribution



$$\tilde{\Delta}_{qq,\text{virt}}^{(1)}(x) \sim \delta(1-x)\Big(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon}\Big) + \text{finite part}$$

  - Real contribution





$$\tilde{\Delta}_{qq,\text{real}}^{(1)}(x) \sim \delta(1-x)\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\frac{1+x^2}{(1-x)_+} - \underbrace{\left[\frac{\ln(1-x)}{(1-x)}\right]_+}_{\text{Plus distribution}}$$

$+$ finite part

The Partonic Cross Section

- The plus distributions
  - Singular functions that have origin in phase space integration
  - Are defined by their integrals with a smooth function (such as parton distribution functions)

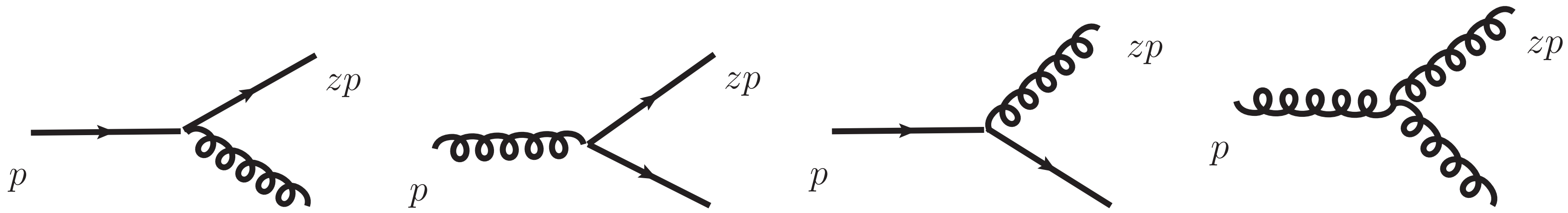
$$\int_0^1 f(x)h_+(x)\mathrm{d}\,x = \int_0^1 h(x)[f(x) - f(1)]\mathrm{d}\,x$$

  - Have to be treated carefuly
- The sum of real and virtual part

$$\tilde{\Delta}_{qq}^{(1)}(x) \quad \sim \quad \underbrace{-\frac{2}{\epsilon} C_f \left[\frac{3}{2}\delta(1-x) + \frac{1+x^2}{(1-x)_+}\right]}_{\text{Splitting function } P_{qq}^{(0)}(x)} + \text{finite part}$$

Splitting Functions

- $P_{ab}(z)$ describes probability of a parton with momentum p radiates a soft or collinear parton



- Perturbative quantities calculated up to NNLO [Moch, Vermaseren, Vogt 2003]

$$P_{ab}^{\mathrm{N}^m\mathrm{LO}}(x,\mu^2) = \sum_{k=0}^{m} \frac{\alpha_s^{k+1}(\mu^2)}{4\pi} P_{ab}^{k}(x)$$

- Main ingredient of the evolution equation of PDFs

The Hadronic cross section

- When the bare parton distribution functions in the cross section are expressed in terms of renormalized ones, in $\overline{\text{MS}}$ scheme

$$
\begin{aligned}
f_{q,0}(x) &= f_q(x,\mu_f^2) - \frac{\alpha_s}{2\pi}\left\{ -\frac{1}{\epsilon} - \ln 4\pi + \gamma_{\mathsf{E}} + \ln\left(\frac{\mu_f^2}{\mu^2}\right)\right\} \\
&\times \left\{ \int_x^1 \frac{\mathrm{d}\,\xi}{\xi} P_{qq}^{(0)}\left(\frac{x}{\xi}\right) f_0(\xi) + \int_x^1 \frac{\mathrm{d}\,\xi}{\xi} P_{qg}^{(0)}\left(\frac{x}{\xi}\right) f_{g,0}(\xi)\right\},
\end{aligned}
$$

the convolution with the divergent partonic cross section

$$
\tilde{\Delta}_{qq}^{(1)}(x) = 2P_{qq}^{(0)}(x)\left[-\frac{1}{\epsilon} - \ln 4\pi + \gamma_{\mathsf{E}} + \ln\left(\frac{\mu^2}{Q^2}\right)\right] + \text{finite part}
$$

leads to finite hadronic cross section as a result of factorization theorem.

$$
\Delta_{qq}^{(1)}(x,\mu_f^2) \equiv \text{finite part} - 2P_{qq}^{(0)}(x)\ln\left(\frac{\mu_f^2}{Q^2}\right)
$$

Parton Distribution Functions

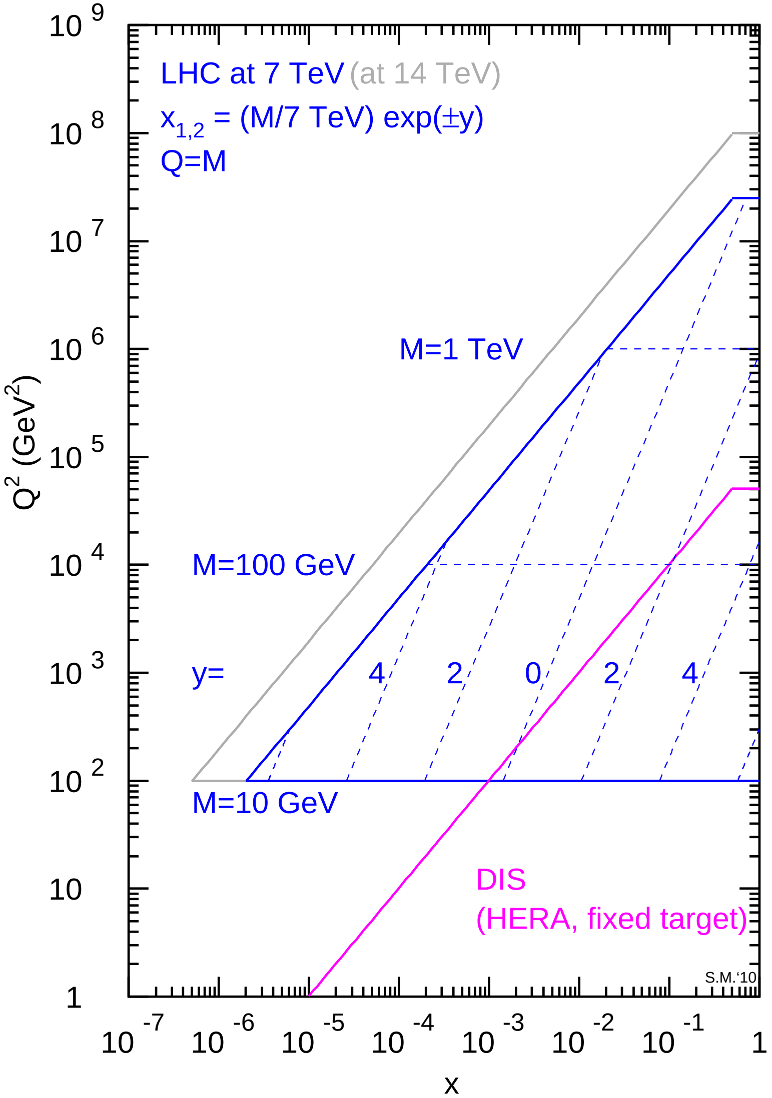
$$i f_a(x, \mu_f^2) \qquad a = q, \bar{q}, g \qquad x \in (0, 1)$$

- Distribution of momentum fraction x of the parton a in the proton
- Non-perturbative objects
- Extraction from experiment (DIS) at low scale $Q_0^2 \sim$ few GeV^2 $\rightarrow$ fitting
- Scale dependence described by evolution equations (DGLAP)

$$\frac{\partial f_a(x, \mu_f^2)}{\partial \ln \mu_f^2} = \sum_b \int_x^1 \frac{\mathrm{d}\,\xi}{\xi} \underbrace{P_{ab}(x/\xi, \mu_r^2)}_{\text{splitting functions}} \quad f_b(\xi, \mu_f^2)$$

- Solution - system of integro–differential equations
- Universal - process independent quantities

Kinematics of PDFs



- Rapidity of the vector boson in CM frame of partons

$$y = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$$

$$\Rightarrow x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}$$

[S.Moch]

The Hadronic Cross Section

$$\frac{\mathrm{d}\,\sigma_{pp\to V}(Q^2)}{\mathrm{d}\,Q^2} = \sum_{a,b}\int_0^1 \mathrm{d}\,x_1 \int_0^1 \mathrm{d}\,x_2\, f_a(x_1,Q^2) f_b(x_2,Q^2) \frac{\mathrm{d}\,\hat{\sigma}^V_{ab}(x_1,x_2,Q^2)}{\mathrm{d}\,Q^2}$$

$$= \mathcal{N}^V \tau \sum_{a,b} \mathcal{C}^V (f_a \otimes f_b \otimes \Delta^V_{ab})(\tau); \qquad Q = \mu_f = \mu_r$$

$$\frac{\mathrm{d}\,\hat{\sigma}}{\mathrm{d}\,Q^2} = \mathcal{N}^V \mathcal{C}^V \Delta_{ab}(x) \qquad \int_0^1 \mathrm{d}\,x\,\delta\Big(x - \frac{\tau}{x_1 x_2}\Big) = 1 \qquad \tau = \frac{Q^2}{s}$$

- Evolution of PDFs

$$\frac{\partial f_a(x,Q^2)}{\partial \ln Q^2} = \sum_b \int_x^1 \frac{\mathrm{d}\,\xi}{\xi} P_{ab}(x/\xi,Q^2) f_b(\xi,Q^2) \frac{\partial f_a(x,Q^2)}{\partial \ln Q^2} = \sum_b (P_{ab} \otimes f_b)(x)$$

Convolution

- Definition

$$
\begin{aligned}
&(f_1 \otimes f_2 \otimes \cdots \otimes f_k)(x) \\
&= \int_0^1 \mathrm{d}\, x_1 \int_0^1 \mathrm{d}\, x_2 \ldots \int_0^1 \mathrm{d}\, x_k \, \delta(x - x_1 x_2 \ldots x_k) f_1(x_1) f_2(x_2) \ldots f(x_k)
\end{aligned}
$$

Convolution and The Mellin Transform

- The Mellin transform

$$\mathbf{M}[f(x)] = \int_0^1 \mathrm{d}\,x\,x^{N-1} f(x) = \tilde{f}(N)$$

$$N = c + \rho e^{i\varphi}, \qquad \rho, c \in \mathbb{R}, \qquad \int_0^1 \mathrm{d}\,x\,x^{c-1} f(x) \qquad \text{abs. convergent}$$

- Mellin transform of the convolution is a product of functions in N space

$$\mathbf{M}[(f_1 \otimes f_2 \otimes \cdots \otimes f_k)(x)](N) \rightarrow \prod_{k=1}^{m} \mathbf{M}[f_k](N)$$

- The cross section

$$(f_a \otimes f_b \otimes \Delta_{ab}^V)(\tau) \rightarrow f_a(N) f_b(N) \Delta_{ab}^V(N) \equiv \tilde{W}(N)$$

The Inverse Mellin Transform

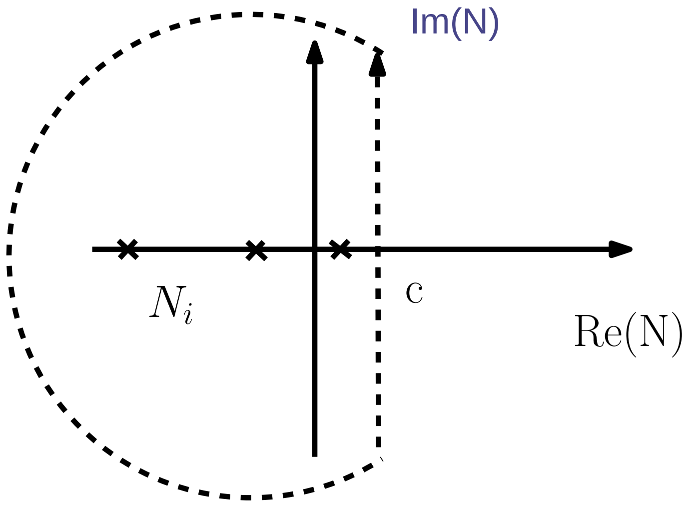
- Inverse Mellin Transform

$$f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\,x^{-N}\,\tilde{f}(N)$$

- The original function $W(\tau)$ can be recovered

$$W(\tau) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\,x^{-N}\,\tilde{W}(N)$$

- $\Rightarrow$ Only one integration



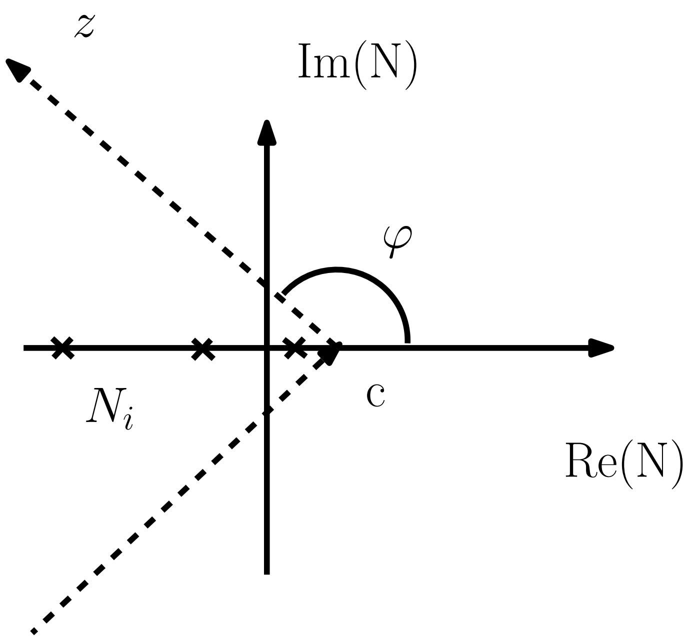
The Mellin Transform - Example

$$
\begin{aligned}
f(x) &= x^2 \\
\tilde{f}(N) &= \int_0^1 \mathrm{d}\,x\, x^2 x^{N-1} f(x) = \frac{1}{N+2} \\
f(x) &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\, x^{-N} \frac{1}{N+2} \\
&= \frac{1}{2\pi i} 2\pi i \frac{x^{-N}(N+2)}{(N+2)} \Big|_{N=-2} = x^2
\end{aligned}
$$

The Inverse Mellin transform

- Numerical evaluation [Vogt 2004]

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\,x^{-N}\tilde{f}(N=c+\rho e^{i\varphi})$$
$$=\frac{1}{\pi}\int_{0}^{\infty} \mathrm{d}\,\rho\,\mathrm{Im}[e^{i\varphi}x^{-c-\rho e^{i\varphi}}f(N)]$$



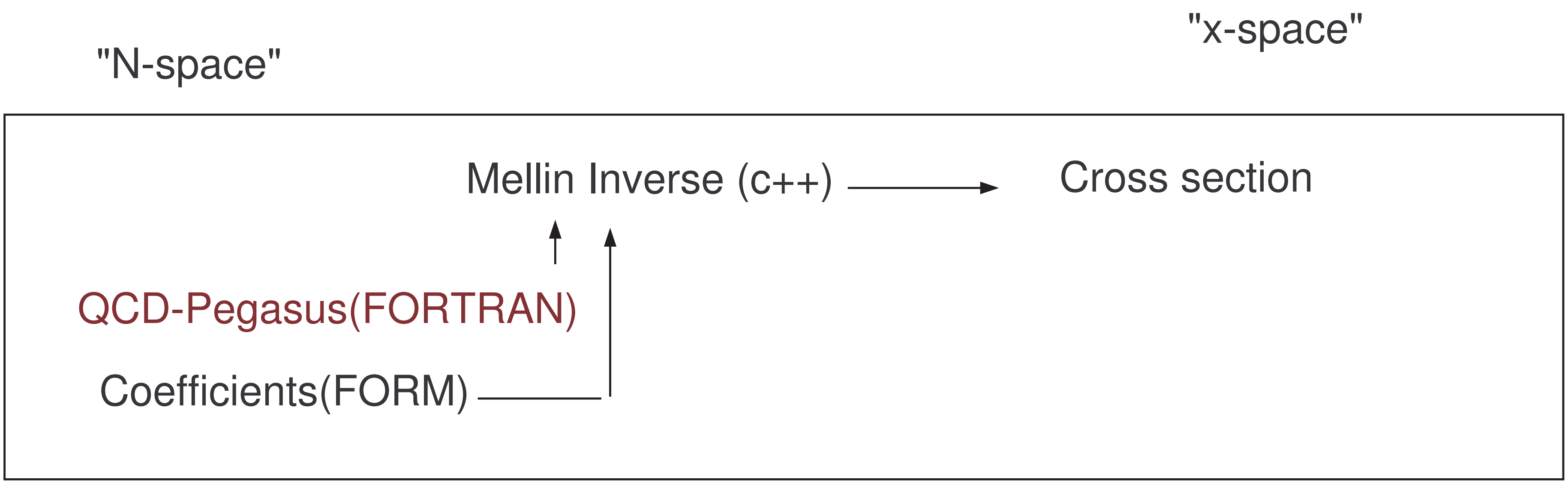
- Integration only up to ρ_{max} due to the factor $\exp[\rho\ln(1/x)\cos\varphi]$
- Parameters φ, c and number of moments N_i can be tuned to get better accuracy
- Mellin transform of coefficient functions and parton distribution functions with an analytic dependence on $N\in\mathbb{C}$ must exist

Mellin Transforms

- Coefficient functions
  - MT's up to next-to-next-to leading order known [Vermaseren 1998; Blümlein, Ravindran 2005]
  - Harmonic polylogarithms $\longrightarrow$ harmonic sums [Vermaseren - harmpol.h (2000), summer.h (1999)]
  - Analytic continuations of harmonic sums in terms of polygamma functions
- PDFs
  - Choose a PDF set and parametrize it at the initial scale $\mu_0^2 \sim$ few GeV^2

$$x f(x, \mu_0^2) \sim x^a (1-x)^b \longrightarrow \underbrace{\beta(n+a, b+1)}_{\text{Euler beta function}}$$

The Calculation - Toy PDFs



- PDFs $\Leftarrow$ QCD-Pegasus [Vogt 2004]
- Coefficient functions - transformation of x space results using using harmpol.h, summer.h [Hamberg, Matsuura, van Neerven 1990; Vermaseren 1998]
  - direct extraction of N space coefficients [Blümlein, Ravindran 2005]
- N space integration using 32 point Gaussian quadrature

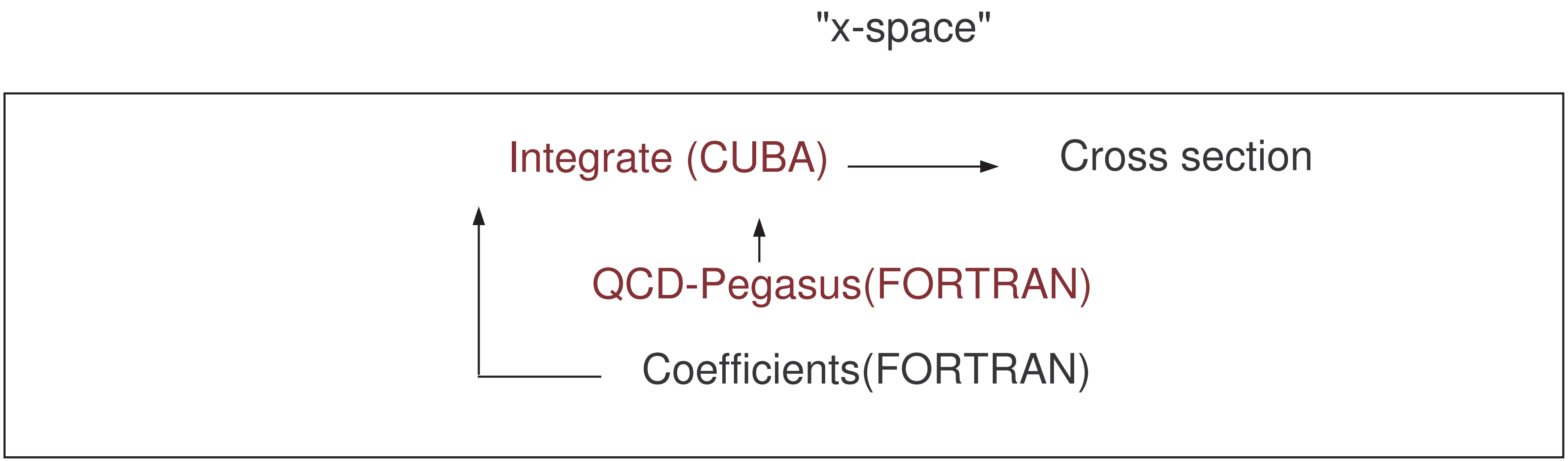
QCD Pegasus

- FORTRAN package performing evolution of polarized and unpolarized evolution of PDFs up to NNLO using Mellin space technique [Vogt 2004]
- Output in both N space and x space
- Two possible input parametrizations

$$
\begin{aligned}
x f_i(x, \mu_0^2) &= \mathcal{N}_i p_{i,1} x^{p_{i,2}} (1-x)^{p_{i,3}} (1 + p_{i,5} x^{p_{i,4}} + p_{i,6} x) \\
x f_i(x, \mu_0^2) &= \mathcal{N}_i p_{i,1} x^{p_{i,2}} (1-x)^{p_{i,3}} (1 + p_{i,4} x^{0.5} + p_{i,5} x + p_{i,6} x^{1.5})
\end{aligned}
$$

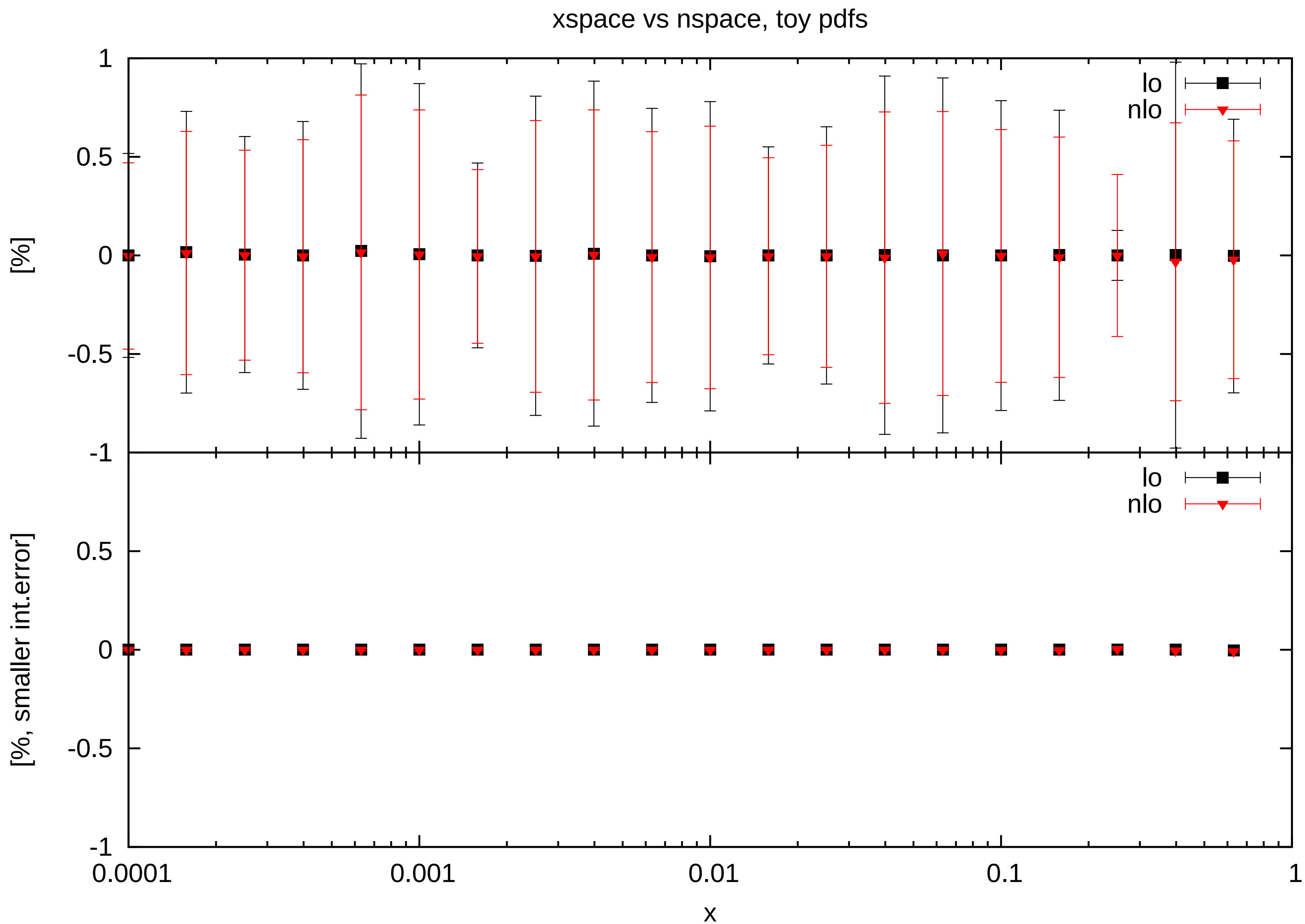
  Defalut values of p_i correspond roughly to the cteq5m PDF set
- Choice of $\mu_f \neq \mu_r$
- VFNS vs FFNS
- `http://www.liv.ac.uk/~avogt/pegasus.html`

The Calculation - Checks (Toy PDFs)

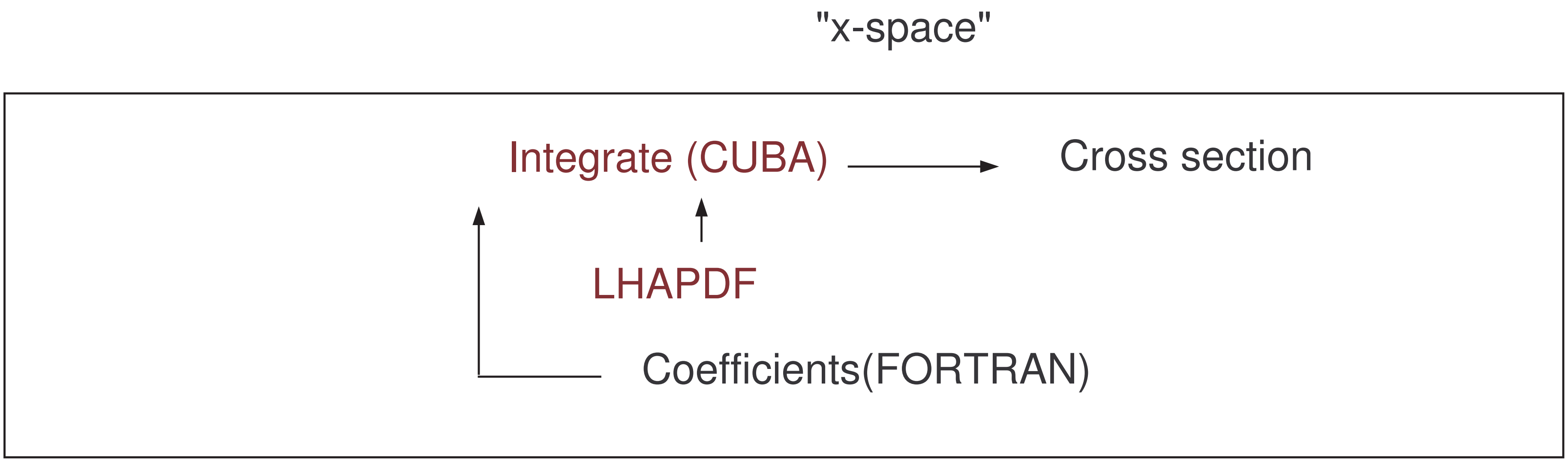


- PDFs $\Leftarrow$ QCD-Pegasus (FORTRAN) [Vogt 2004]
- Coefficient functions in x space [Hamberg, Matsuura, van Neerven 1990]
- x space integration - CUHRE algorithm using CUBA library [Hahn 2005]

Comparison N space vs x space, toy PDFs

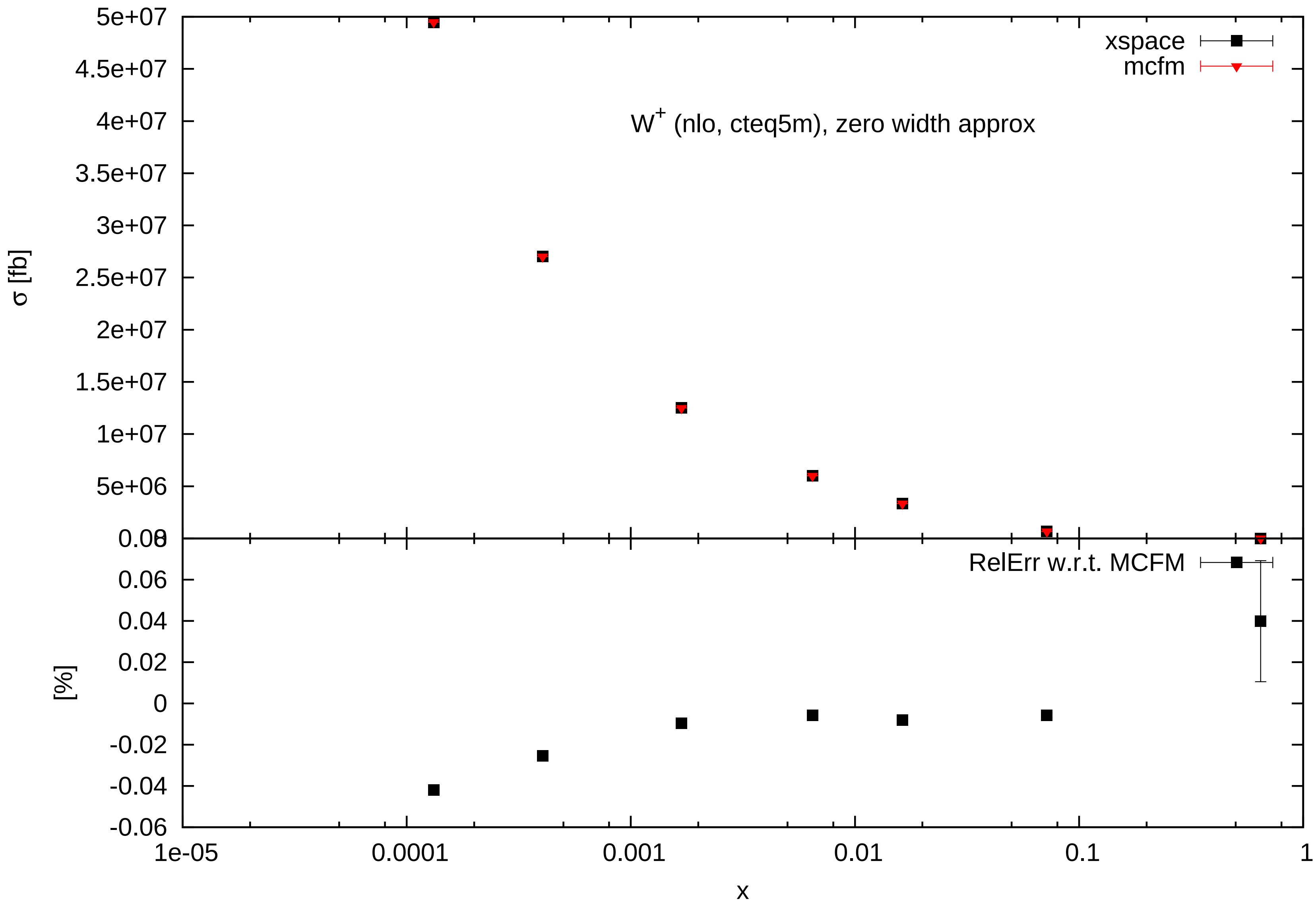


Calculation - Checks, LHAPDF



- PDFs $\Leftarrow$ LHAPDF [HepForge]
- Comparison
  - MCFM [Campbell, Ellis]
  - DYNNLO [Catani, Cieri, Ferrera, de Florian, Grazzini 2009; Catani, Grazzini 2007]

Comparison of x space with MCFM



Discussion

- Speed (no optimization of either code so far)
  Ratio of standard x-space calculation $(\sigma(x))$ vs Mellin space approach $(\sigma_N(x))$ using LHAPDF grid for x-space, required relative error on CUBA integration 10^{-3}.

$$x = 10^{-2} \qquad \sigma(x)/\sigma_N(x) \sim 10^2$$
$$x = 10^{-4} \qquad \sigma(x)/\sigma_N(x) \sim 10^3$$

- Accuracy
  relative error of nspace and xspace in the $\sim$ error on x space integration
- No need of special care of plus distributions in N space
- Coefficient functions in N space for other processes (DIS, Higgs production) are availiable
- PDFs need improvement
  - Restricted by the functional form
  - Using different PDF sets requires user to do own parametrization, functions with more parameters may be necessary

Summary and Outlook

- Higher order corrections for Drell-Yan process are necessary for precise predictions
- Calculation in Mellin space is fast and accurate

- NNLO DY, possible parametrization
- Deep Inelastic Scattering
- Release the code

Backup

- The plus distribution

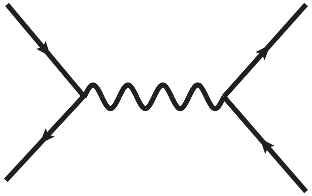
$$(1-x)^{-1-2\epsilon} = -\frac{\delta(1-x)}{2\epsilon} + \frac{1}{(1-x)_+} - 2\epsilon\left[\frac{\ln(1-x)}{1-x}\right]_+ + \mathcal{O}(\epsilon^2)$$

- Analytic continuation of a Harmonic Sum

$$S_k(N) = \frac{(-1)^{k+1}}{(k-1)!}\psi^{k-1}(N+1) + \zeta(k) \qquad (1)$$

The Partonic Cross Section

- Leading order



  - W exchange

$$\frac{\mathrm{d}\,\hat{\sigma}^{W}_{q_k q_l \to l\bar{l}}}{\mathrm{d}\,Q^2} = \underbrace{\frac{\pi\alpha}{M_W \sin\theta_W{}^2 N_c}\,\frac{\Gamma_{W\to l\bar{\nu}}}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2}}_{\mathcal{N}^W}\,\underbrace{|V_{q_k q_l}|^2}_{\mathcal{C}^W}\,\underbrace{\delta(1-x)}_{\Delta^{(0)}_{qq}(x)}$$

  - $\Delta^{(0)}_{qq}(x) = \delta(1-x)$