Universal scaling laws for QCD with many flavors

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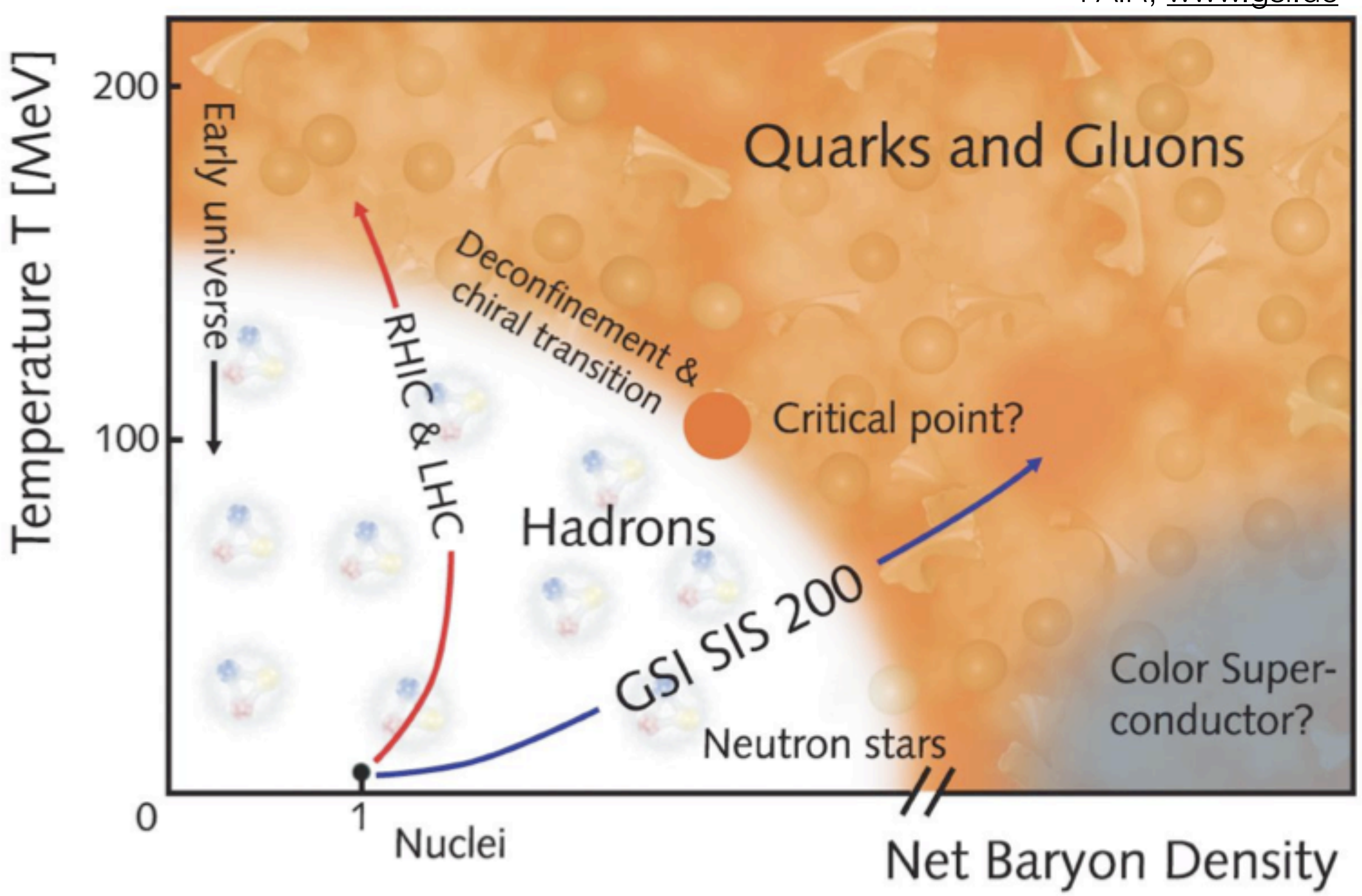
FAIR, www.gsi.de



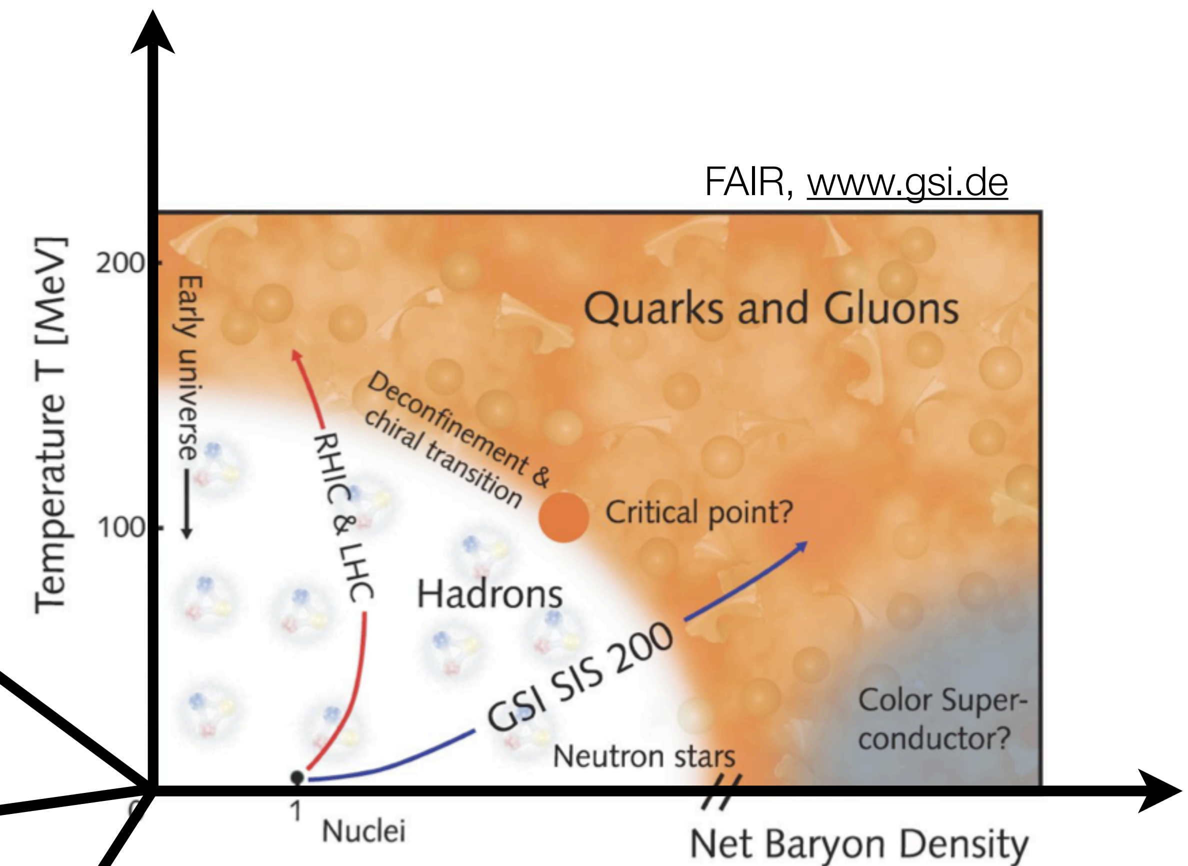
“Learning by deforming”

Many-flavor QCD

weak-coupling limit

Many-color QCD

...

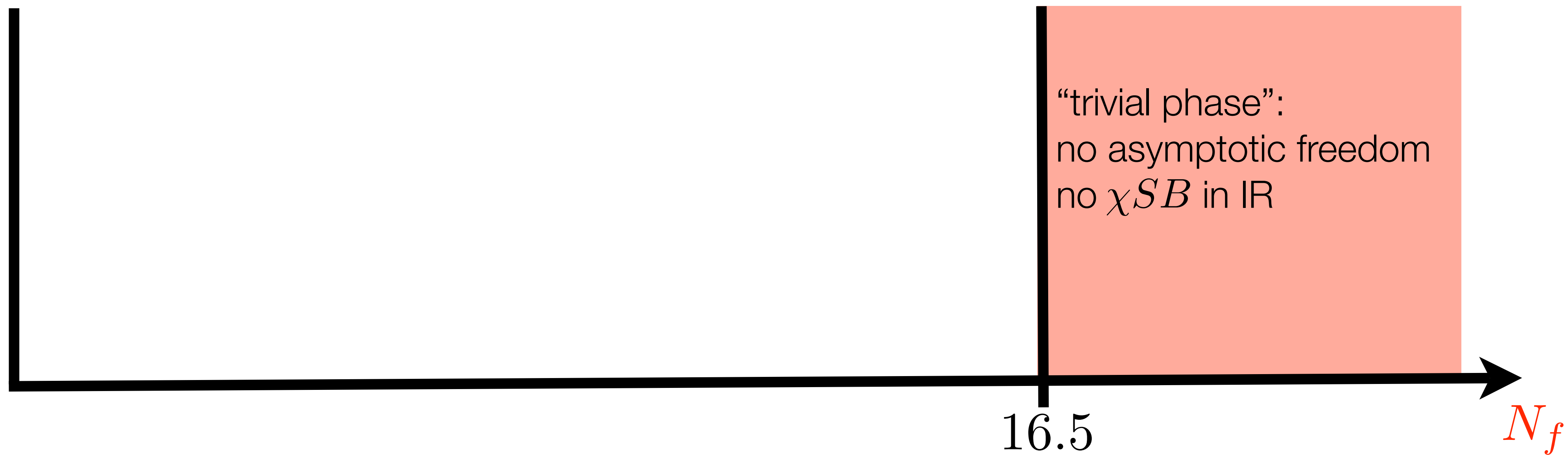


$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \left(\mathrm{i} \partial\!\!\!/ + \mathrm{i} g A\!\!\!/ \right) \psi$$

symmetries:

$$SU(N_c) \times SU_L(N_f) \times SU_R(N_f)$$

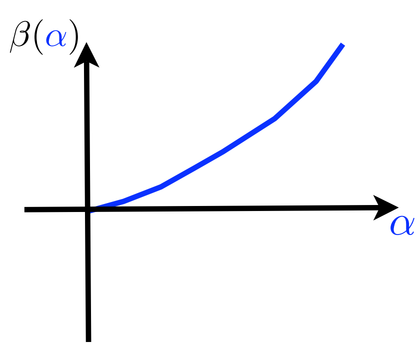
Many flavor QCD at vanishing temperature



- one-loop β-function

$$\partial_t \frac{g^2}{4\pi} = \partial_t \alpha \equiv \beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11N_c - 2N_f)}^{b_1}\,\alpha^2$$

- $b_1 < 0 \Longrightarrow N_f > \frac{11}{2}N_c \overset{N_c=3}{=} 16.5$ (QCD is NOT asymptotically free)



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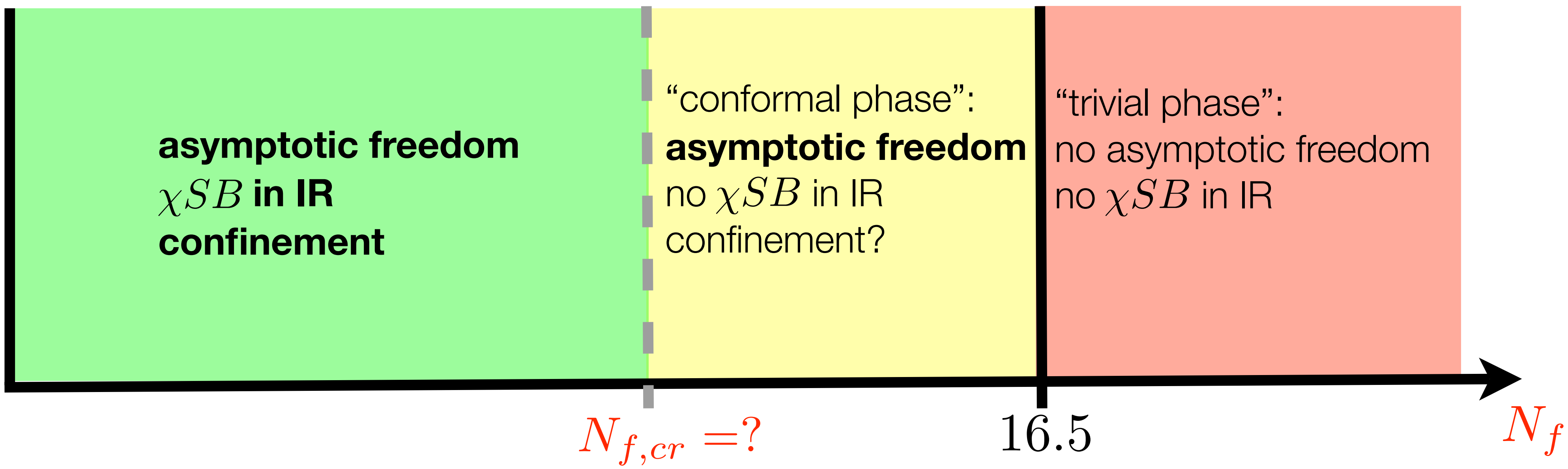

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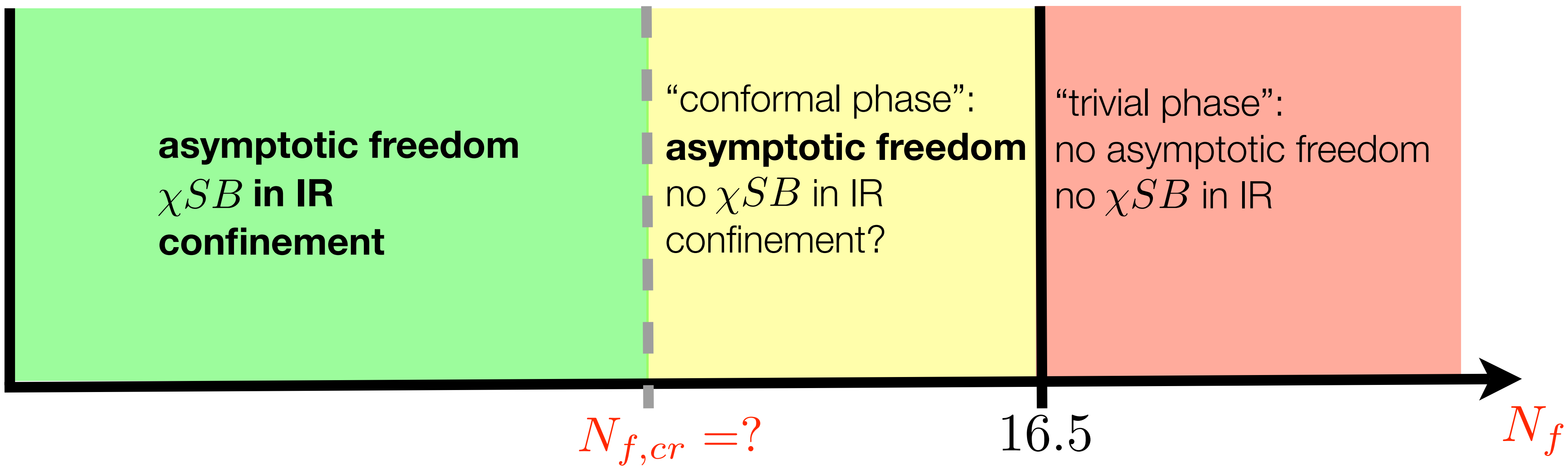
- $b_1 > 0$: QCD is asymptotically free

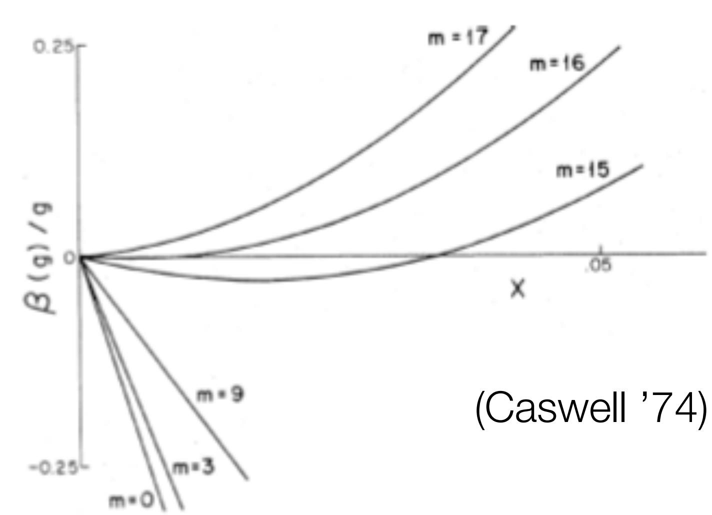
Many flavor QCD at vanishing temperature



- two-loop β-function

$$\partial_t \alpha \equiv \beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11N_c - 2N_f)}^{b_1}\,\alpha^2 - \overbrace{\frac{1}{8\pi^2}\left(\frac{34N_c^3 + 3N_f - 13N_c^2 N_f}{3N_c}\right)}^{b_2}\,\alpha^3$$

- non-trivial infrared fixed point α_* for $8.05 \lesssim N_f < 16.5$
  (Caswell '74; Banks & Zaks '82)



(Caswell '74)

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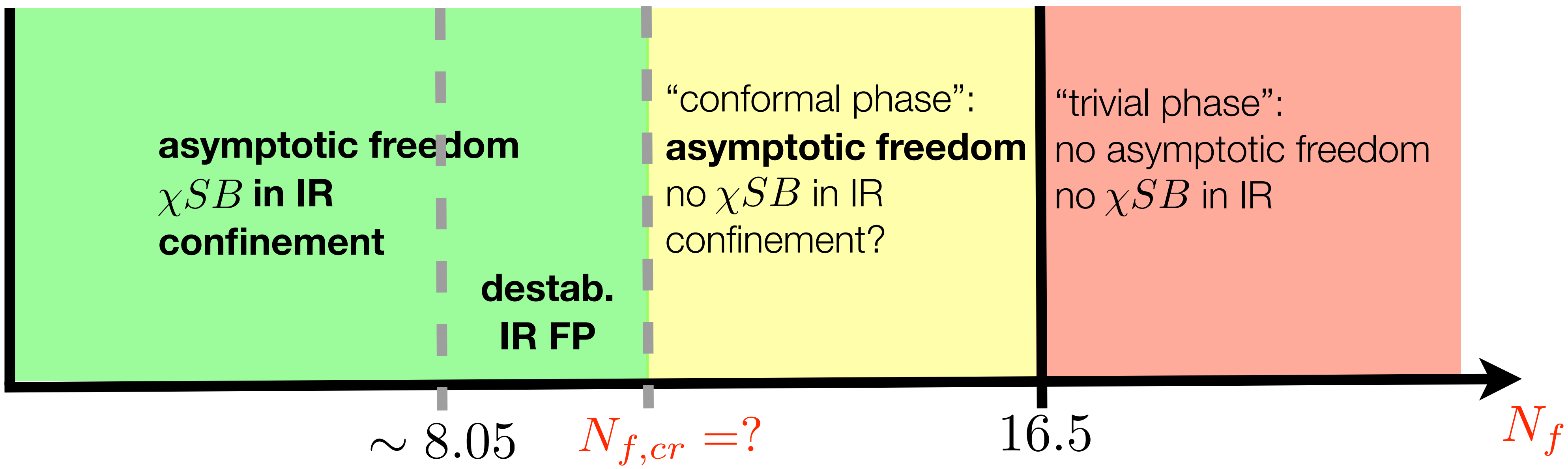
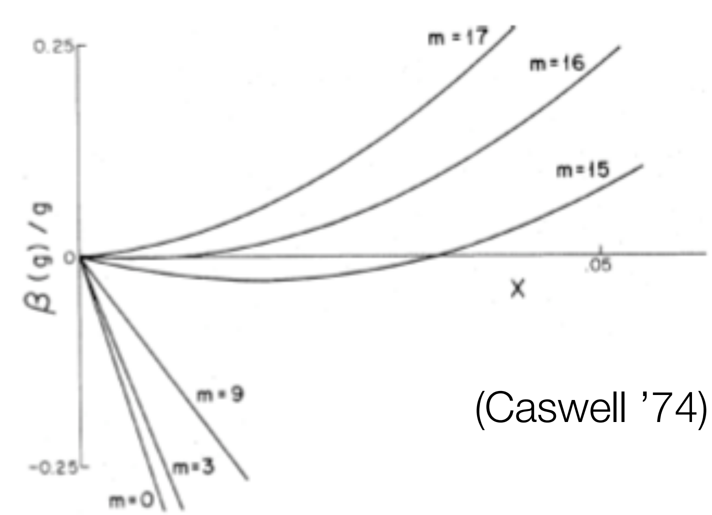


- Caswell-Banks-Zaks fixed gets destabilized due to **chiral symmetry breaking:**

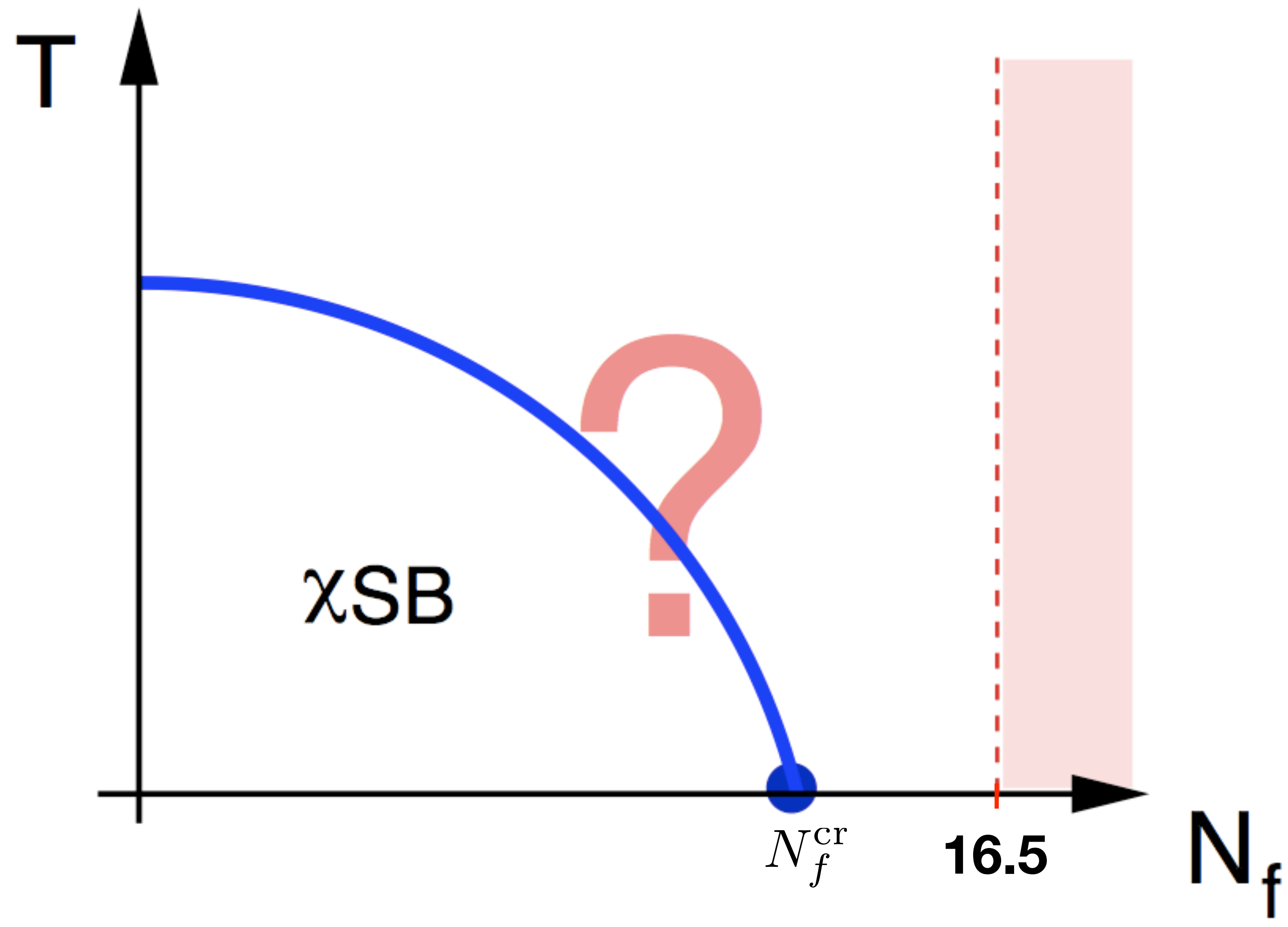
$$g^2 > g^2_{\text{cr}} \text{ : fermions acquire mass, i. e. } N_f^{\text{eff.}} \to 0$$

- cf. quantum phase transition in 3d QED (R. D. Pisarski '84)



(Caswell '74)

Many-flavor phase diagram of QCD



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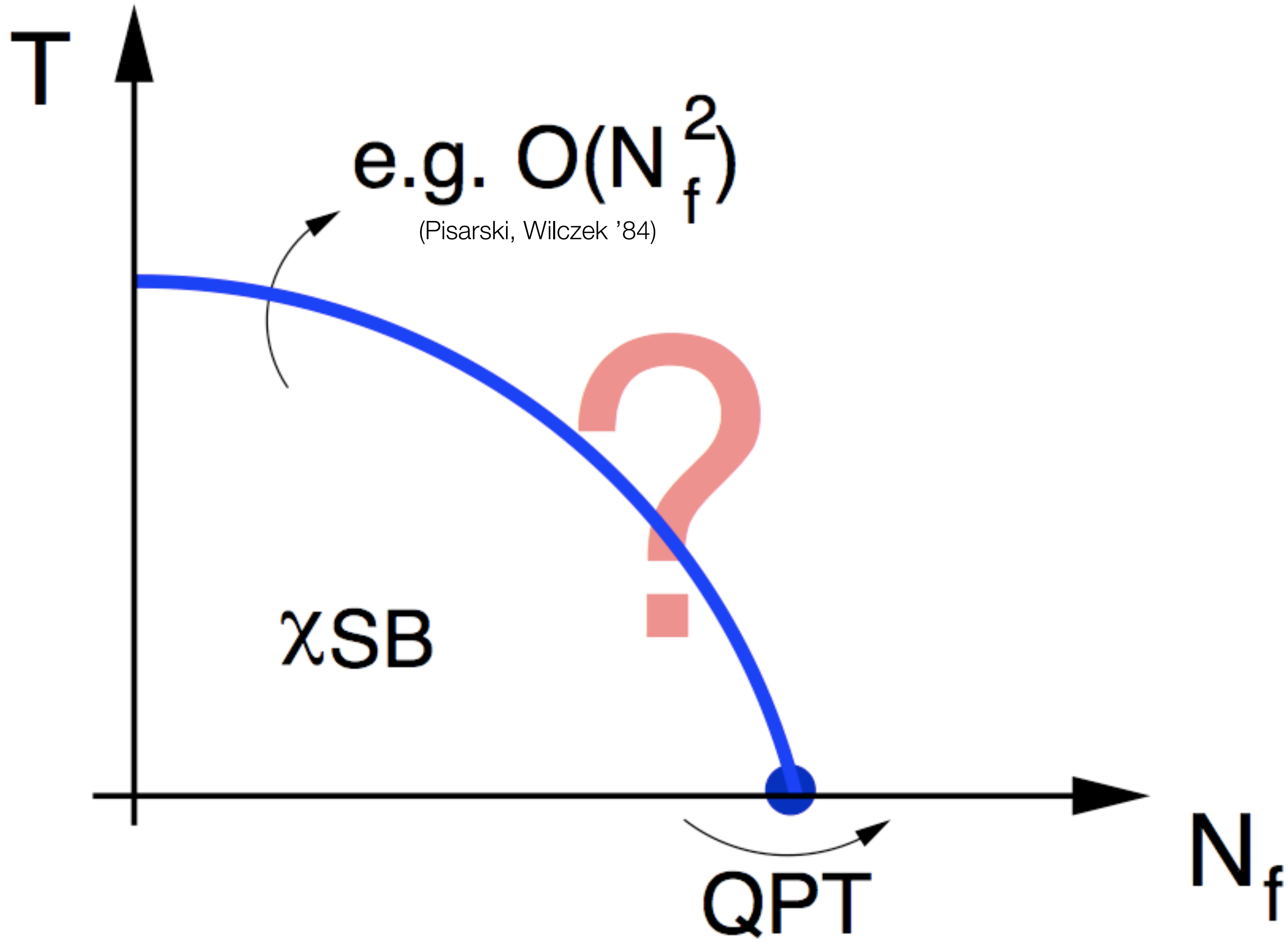
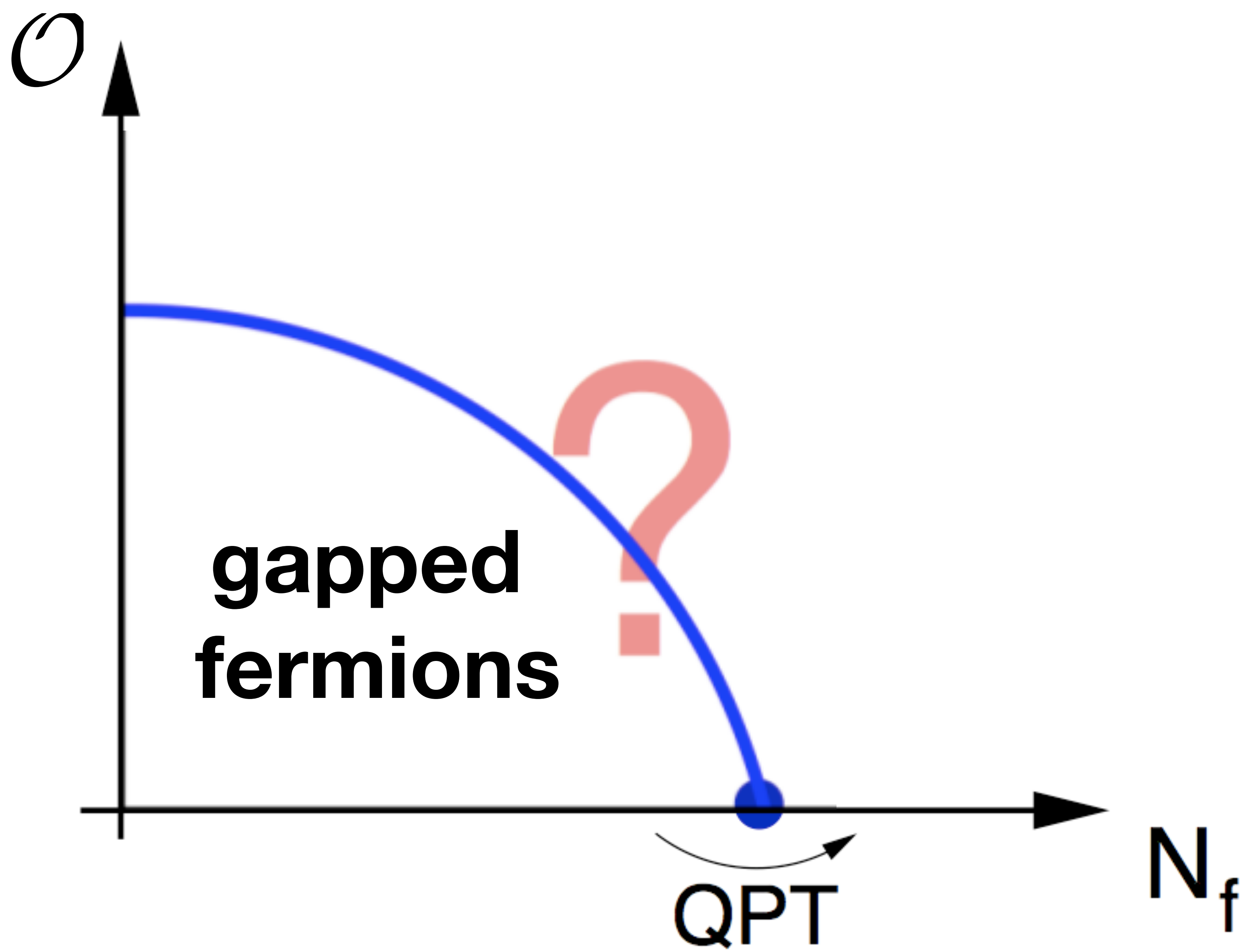


Strongly-flavored gauge theories in general ...



scaling of observables $\mathcal{O}$ in gauge theories with many fermions, such as QED_3, QCD, $\ldots$?

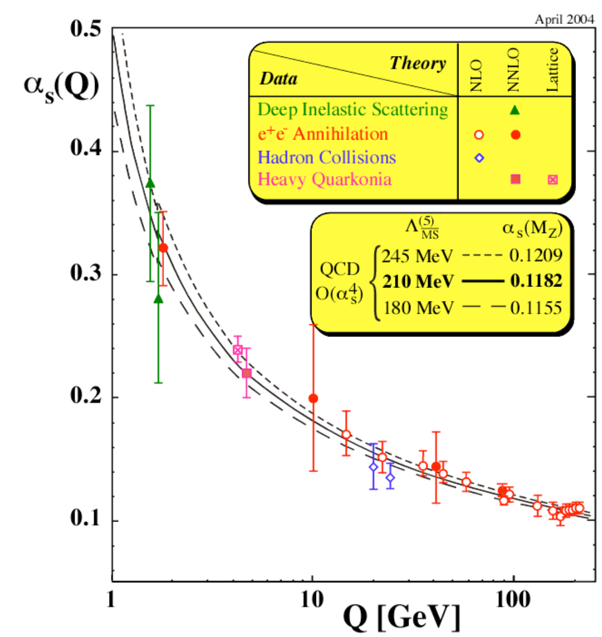
Shape of the phase boundary: **Small** $N_f \ll N_{f,\mathrm{cr}}$

- scale dependence of observables in the chiral limit:

$$T_{\chi\mathrm{SB}},\ f_\pi,\ |\langle\bar\psi\psi\rangle|^{\frac{1}{3}},\ \cdots \sim \Lambda_{\mathrm{QCD}}$$

- position of the Landau pole $\sim \Lambda_{\mathrm{QCD}}$

$$\partial_t \alpha \equiv \beta(\alpha) = -\frac{1}{6\pi}(11N_c - 2N_f)\alpha^2$$



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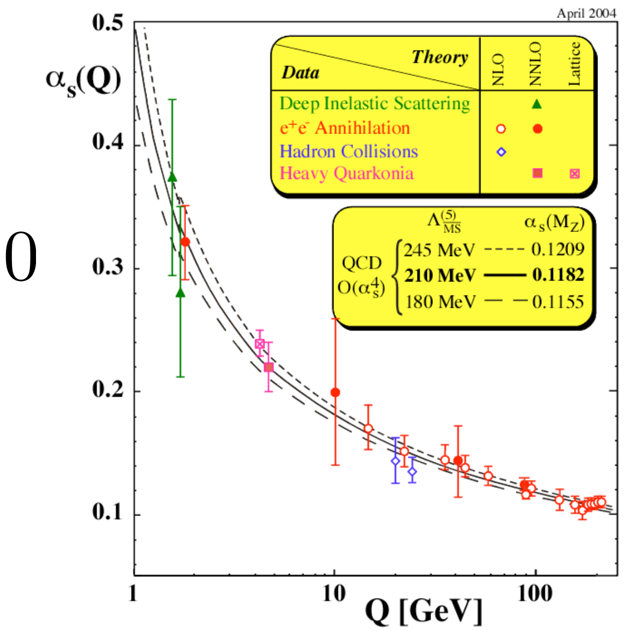
$$\frac{1}{\alpha(\mu_0)} + 4\pi b_0 \ln\frac{\Lambda_{\mathrm{QCD}}}{\mu_0} = \frac{1}{\alpha(\Lambda_{\mathrm{QCD}})} \longrightarrow 0$$

$$\text{with}\quad b_0 = \frac{1}{8\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$



perturbative RG scale: $\mu_0 = m_\tau, m_Z, \ldots$

- ensure comparability of different theories, e. g., by using

$$\alpha(m_\tau) \text{ for all } N_f$$

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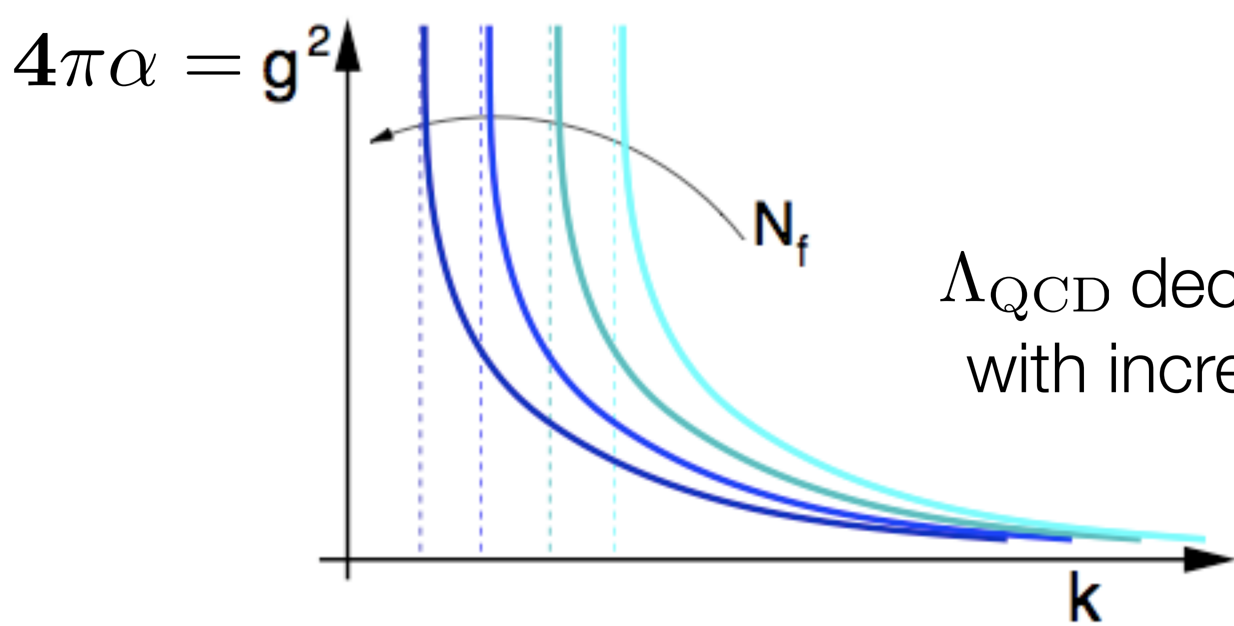


Λ_{QCD} decreases with increasing N_f

Shape of the phase boundary: **Small** $N_f \ll N_{f,\mathrm{cr}}$

(JB, H. Gies '05,'06)

- scale dependence of observables in the chiral limit:

$$T_{\chi\mathrm{SB}},\ f_\pi,\ |\langle\bar\psi\psi\rangle|^{\frac{1}{3}},\ \dots \sim \Lambda_{\mathrm{QCD}}$$

- scaling of $T_{\chi\mathrm{SB}}$ for small N_f

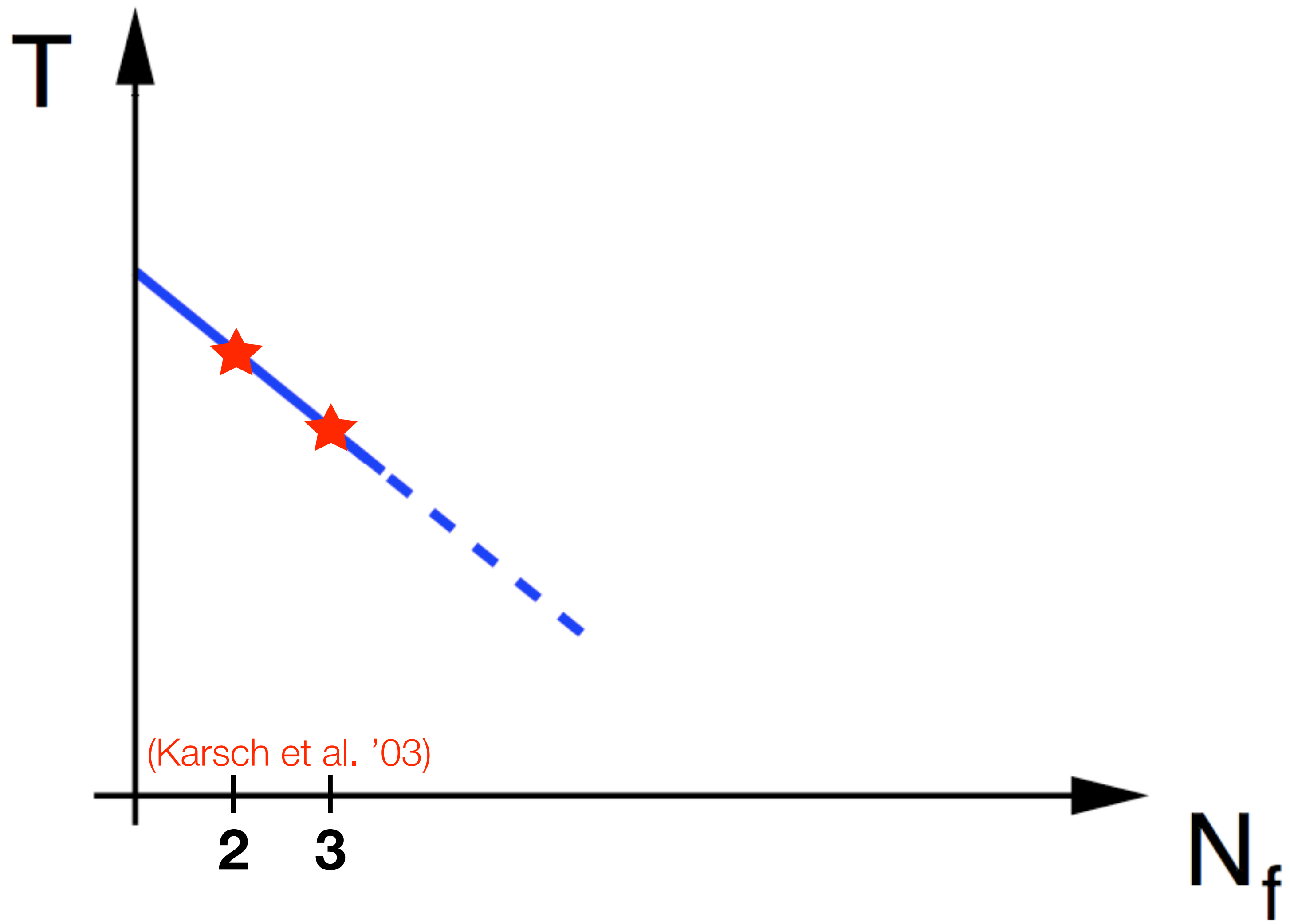
$$\begin{aligned} T_{\chi\mathrm{SB}} &\sim \Lambda_{\mathrm{QCD}} \simeq \mu_0 \mathrm{e}^{-\frac{1}{4\pi\alpha(\mu_0)}} \\ &\simeq \mu_0 \mathrm{e}^{-\frac{6\pi}{11\mathrm{N_c}\alpha(\mu_0)}} \left(1 - \epsilon\mathrm{N_f} + \mathcal{O}((\epsilon\mathrm{N_f})^2)\right) \end{aligned}$$

with $\epsilon = \dfrac{12\pi}{121 N_c^2 \alpha(\mu_0)} \simeq 0.107 \quad \text{for} \quad N_c = 3 \quad \text{and} \quad \mu_0 = m_\tau$

- $T_{\chi\mathrm{SB}}$ scales linearly for small N_f

Shape of the phase boundary: **Small** $N_f \ll N_{f,\mathrm{cr}}$

(JB, H. Gies '05,'06)

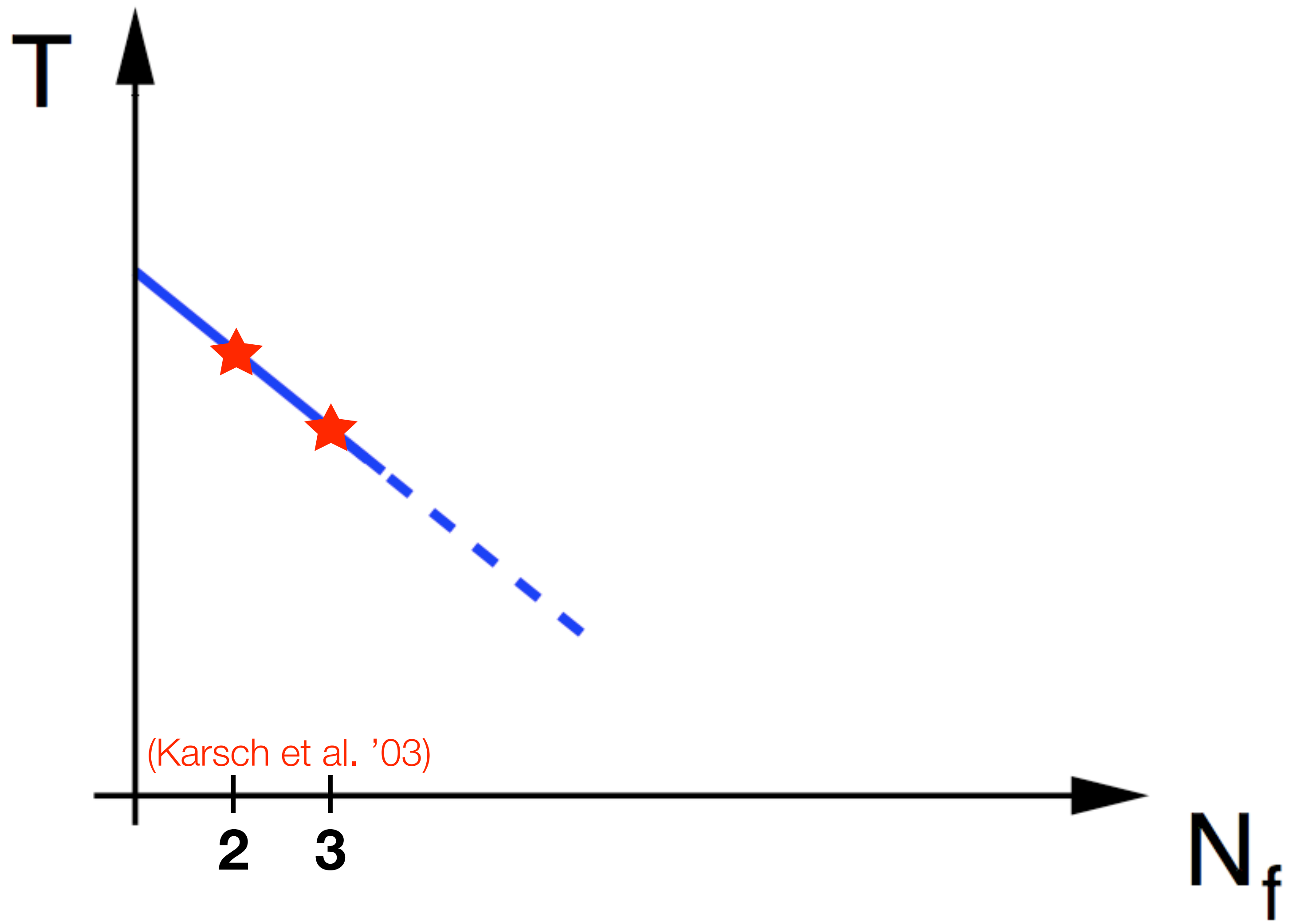


$$\Delta = 2\,\frac{T_{\chi\mathrm{SB}}(2) - T_{\chi\mathrm{SB}}(3)}{T_{\chi\mathrm{SB}}(2) + T_{\chi\mathrm{SB}}(3)} \approx 0.146$$

$(\text{cf. Karsch et al. '03}: \Delta \approx 0.121 \pm 0.069)$

Shape of the phase boundary: **Small** $N_f \ll N_{f,\mathrm{cr}}$

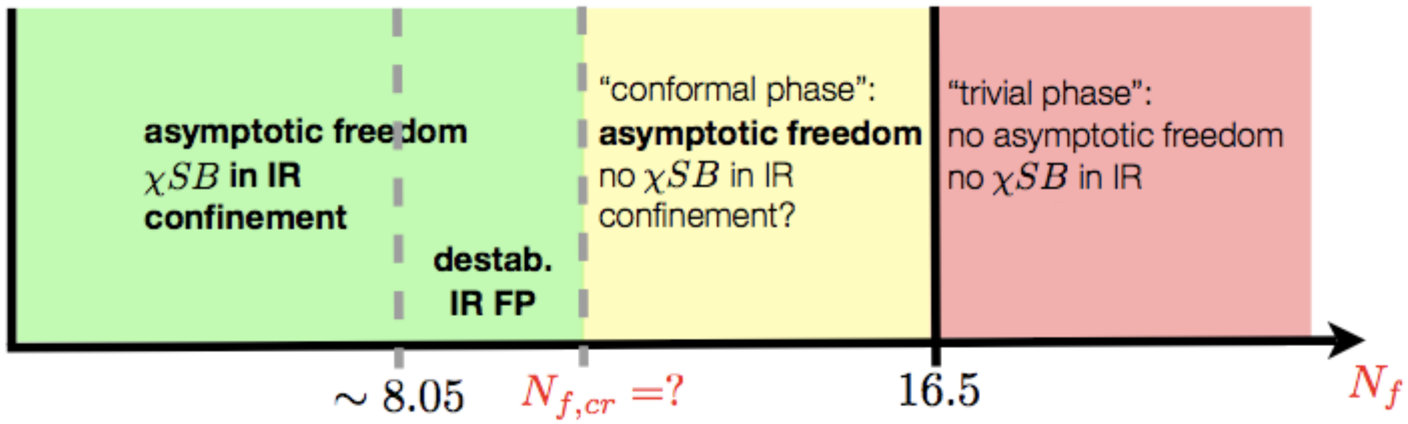
(JB, H. Gies '05,'06)



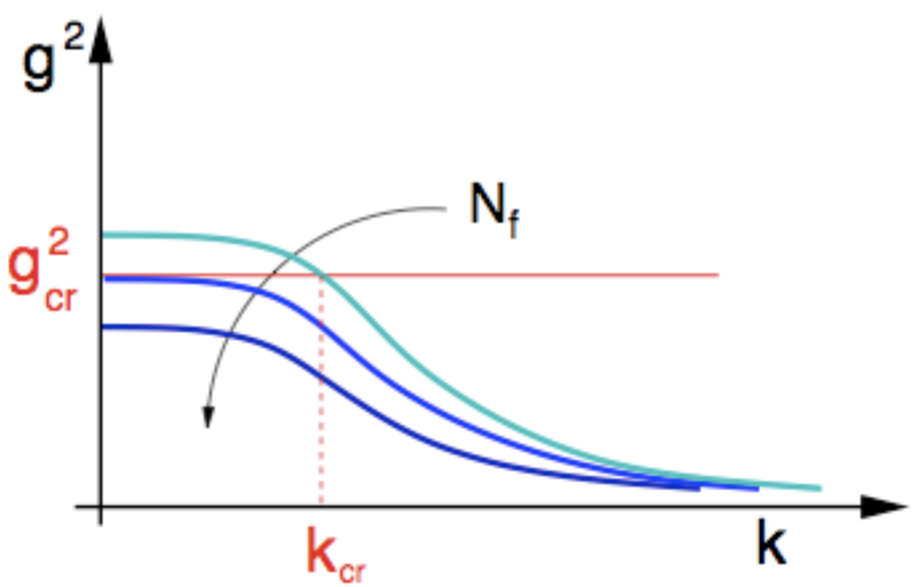
- application: Improved scaling of PNJL/PQM model parameters:
  yields significant improvement of thermodynamics
  (see e. g. Schaefer, Pawlowski, Wambach' 07)

Shape of the phase boundary: **Many** flavors

- lower end of the **conformal window** is determined by the onset of **chiral symmetry breaking**



- **chiral symmetry breaking** requires the strong coupling to exceed a critical value (assumption)



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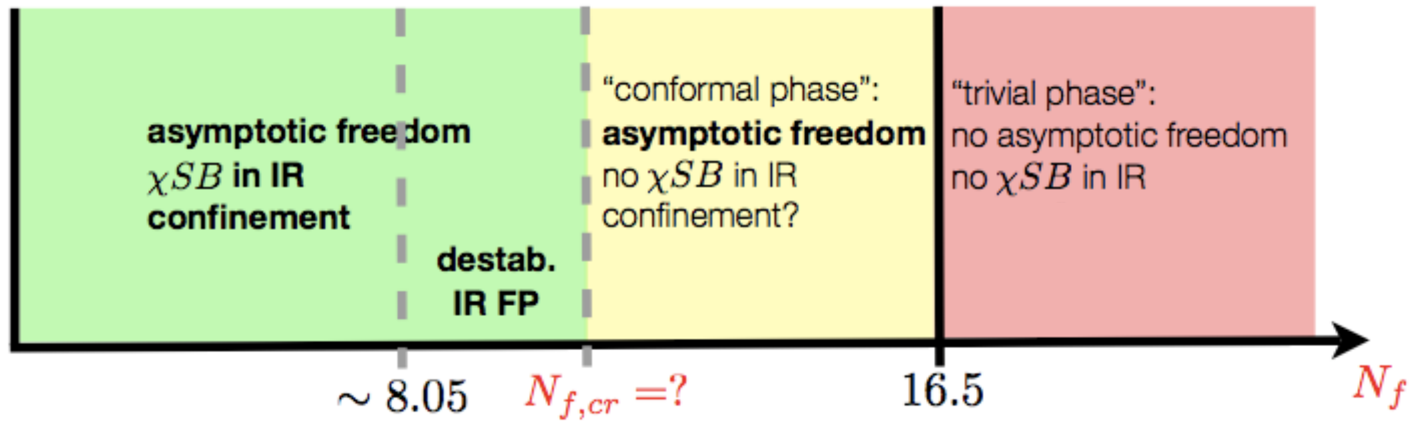
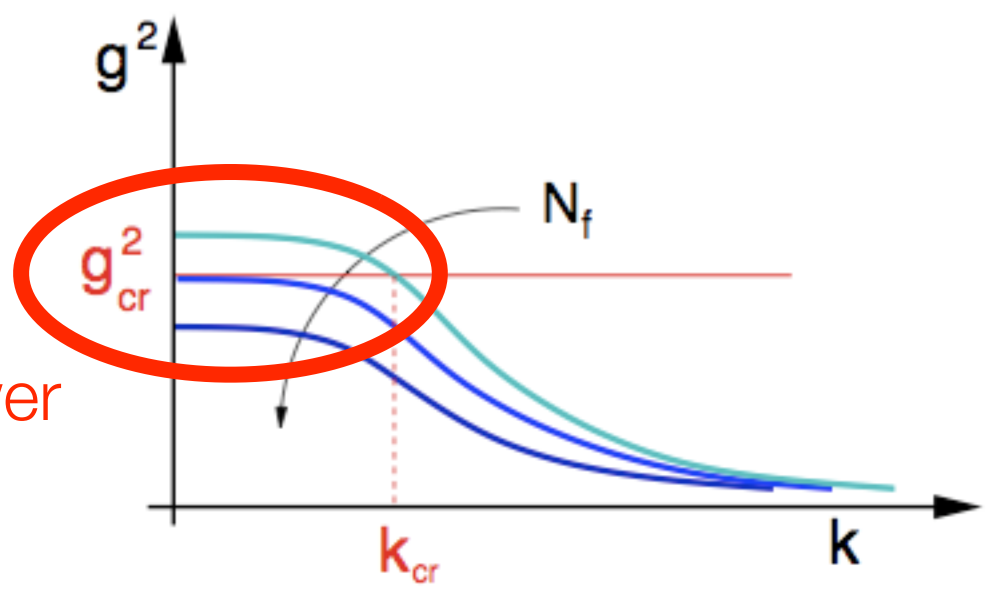


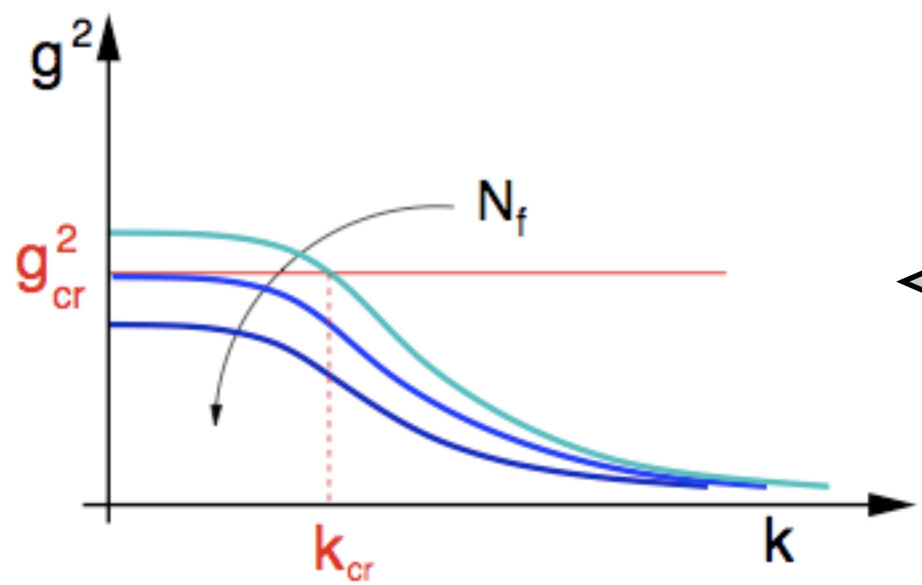
- **chiral symmetry breaking** requires the strong coupling to exceed a critical value (assumption)

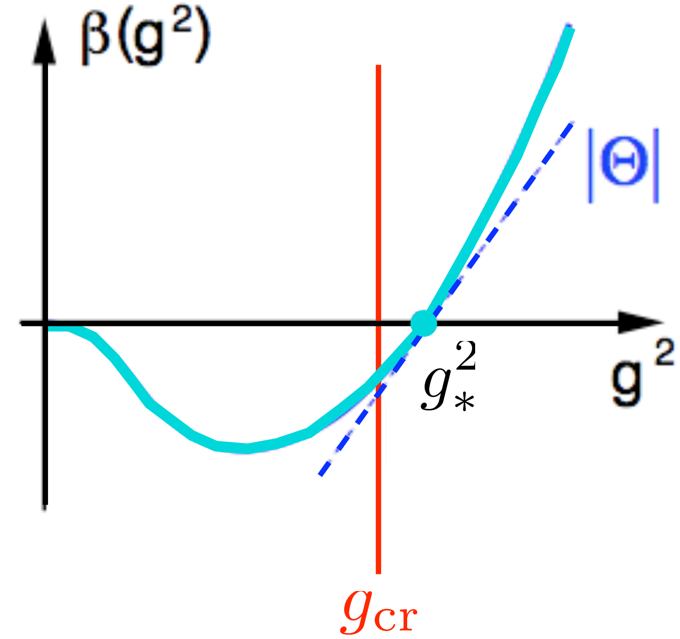
fixed-point regime is relevant close to the lower end of the conformal window



Shape of the phase boundary: Many flavors

(JB, H. Gies '05, '06, '09)





- RG flow in the vicinity of the fixed point g_* is governed by the **universal** critical exponent Θ:

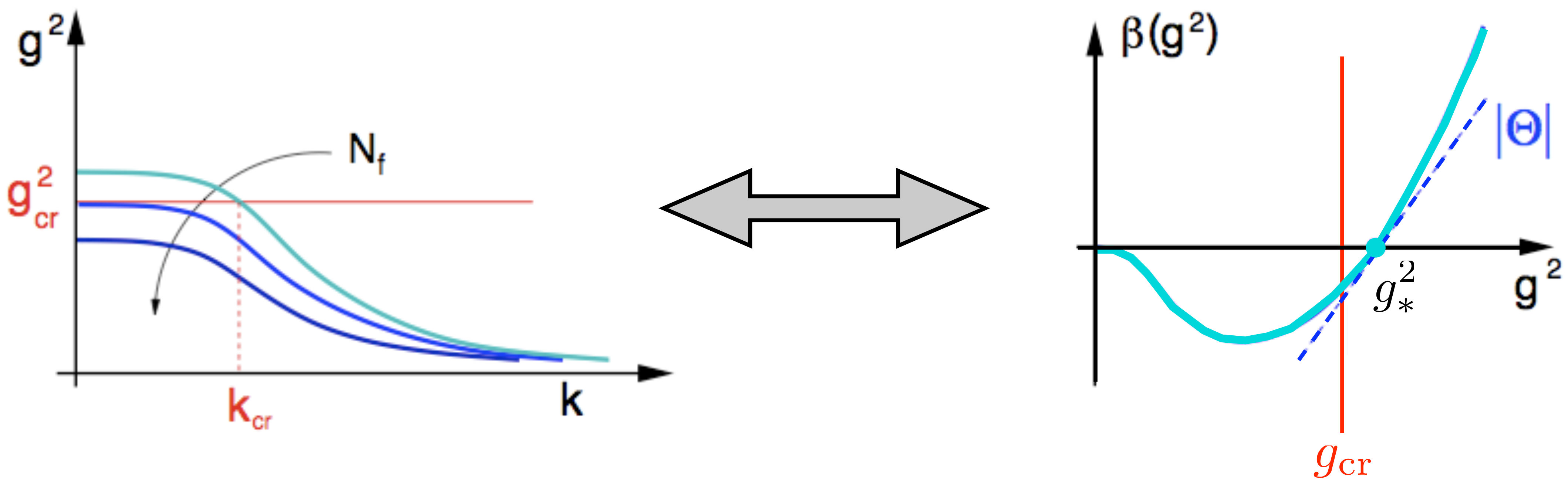
$$k\partial_k g^2 = \beta(g^2) = -\Theta(g^2 - g_*^2) + \ldots$$

- solution in the fixed-point regime:

$$g^2(k) = g_*^2 - \left(\frac{k}{\mu_0}\right)^{|\Theta|}$$

Shape of the phase boundary: Many flavors

(JB, H. Gies '05, '06, '09)



- $g^2(k) \overset{!}{=} g^2_{cr}$: onset of χSB at $k_{cr} \simeq \mu_0 (g^2_* - g^2_{cr})^{\frac{1}{|\Theta|}}$
- scale dependence of observables in the chiral limit:

$$T_{\chi\mathrm{SB}},\ f_\pi,\ |\langle\bar\psi\psi\rangle|^{\frac{1}{3}},\ \dots \sim k_{cr}$$

- proportionality: $g^2_* \sim N_f$

Shape of the phase boundary: Many flavors

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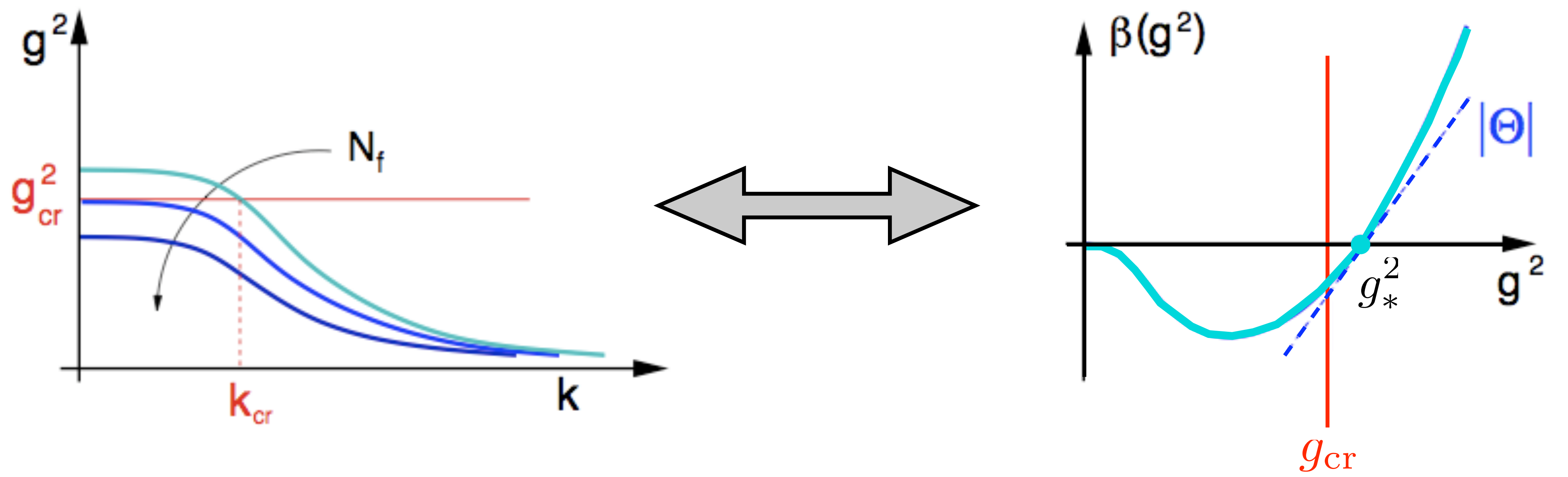

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$$T_{\chi\rm SB},\ f_\pi,\ |\langle \bar\psi \psi \rangle|^{\frac{1}{3}},\ \dots \sim k_{\rm cr}$$

- proportionality: $g_*^2 \sim N_f$
- scaling relation for the critical temperature:

$$T_{\chi\rm SB} \simeq \mu_0 |N_f - N_f^{\rm cr}|^{\frac{1}{|\Theta|}} \quad \text{with} \quad \Theta = \Theta(N_f^{\rm cr})$$

Shape of the phase boundary: Many flavors

(JB, H. Gies '05, '06, '09)

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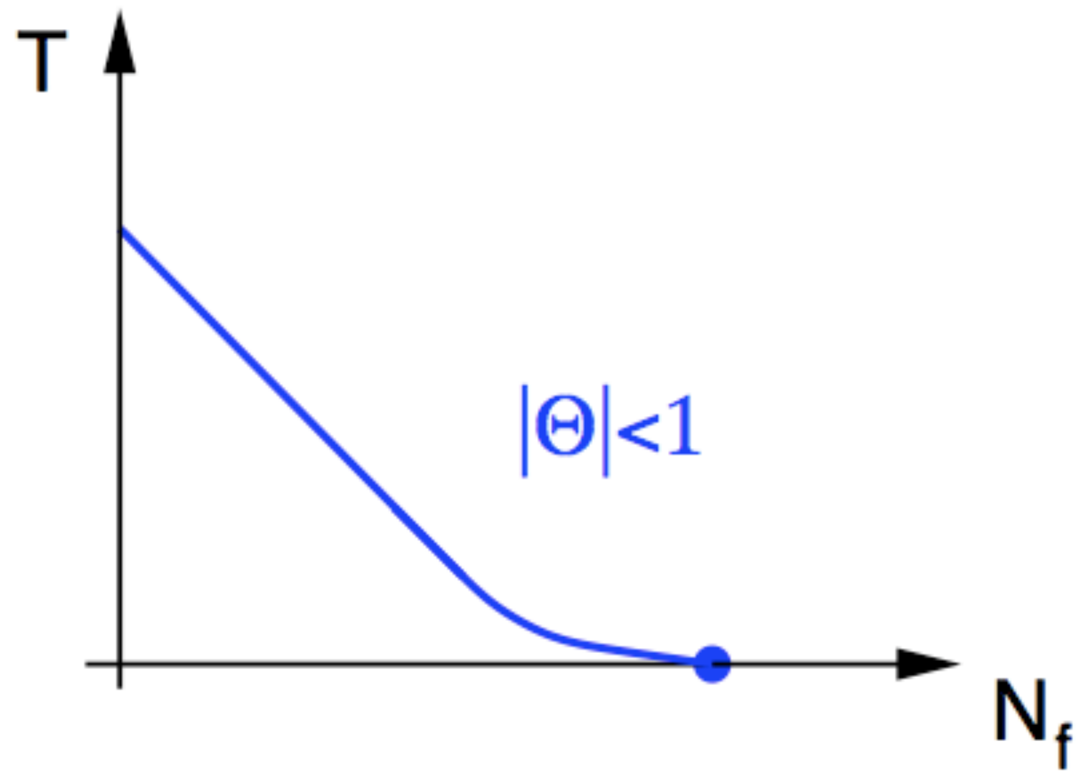
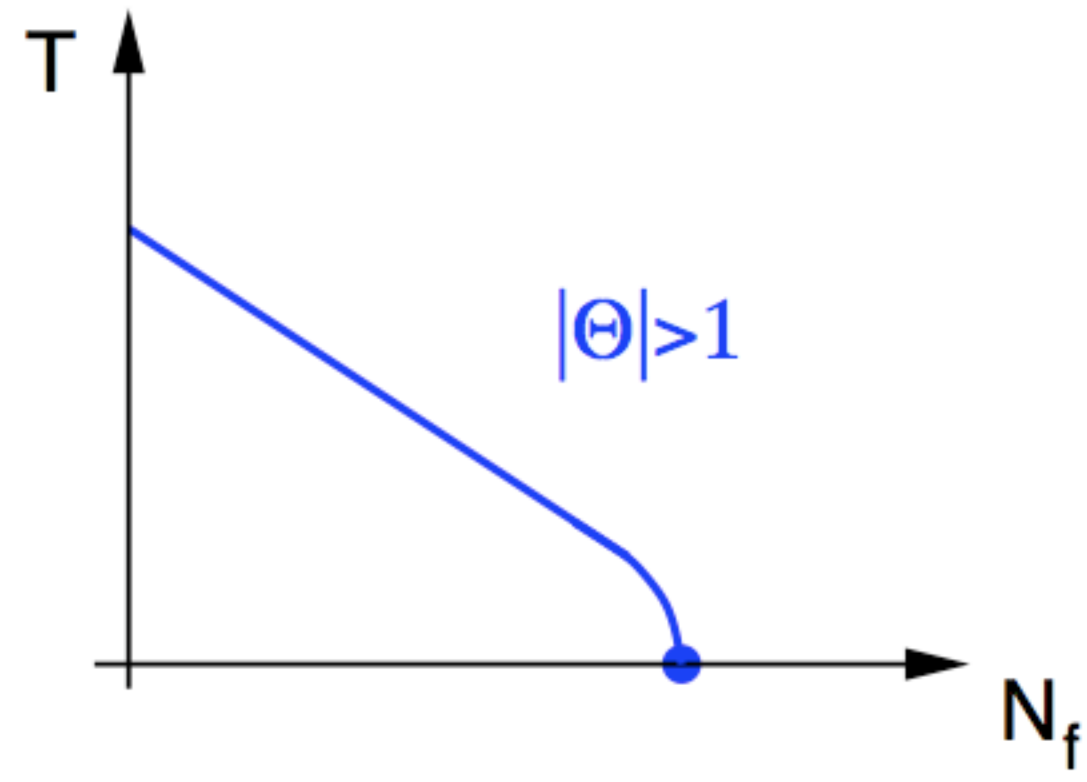
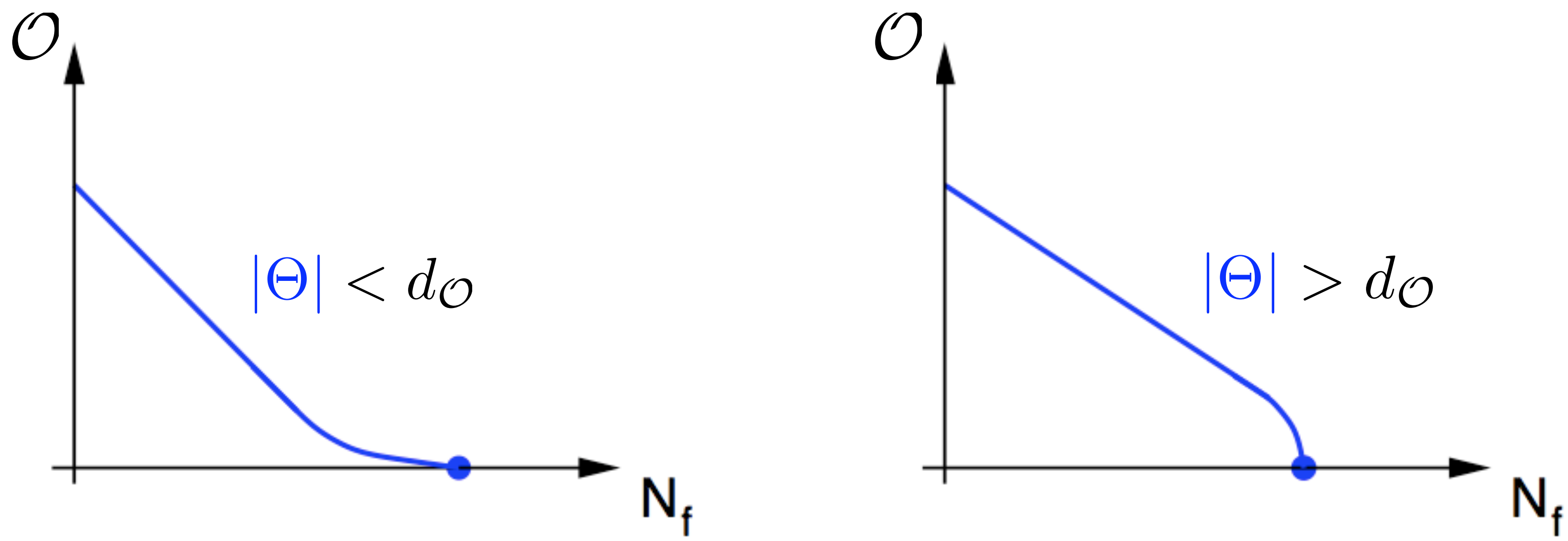




- relation between two **universal** quantities
- relation between **IR gauge dynamics** and **chiral phase structure**
- parameter-free prediction

Scaling of observables near the conformal window

(JB, H. Gies,'09)



- generalization to other (chiral) observables $\mathcal{O}$ with mass dimension $d_{\mathcal{O}}$:

$$\mathcal{O} \simeq \mu_0^{d_{\mathcal{O}}} F_{\mathcal{O}}(N_f)|N_f - N_f^{\mathrm{cr}}|^{\frac{d_{\mathcal{O}}}{|\Theta|}}$$

with $\mathcal{O} = f_\pi, \langle\bar{\psi}\psi\rangle, \ldots$

- dimensionless function $F_{\mathcal{O}}$ depends on N_f but **not** on N_f^{cr}
- power-law-like behavior of the **correlation length** near the **quantum critical point**

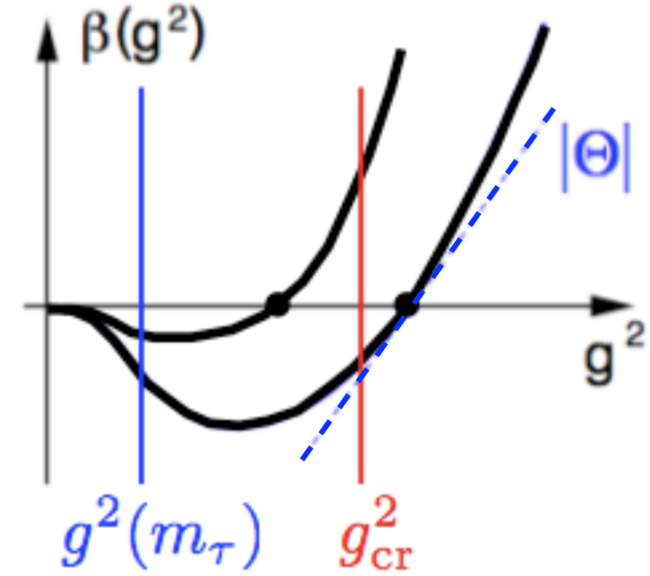
What about Miransky scaling?

(JB, H. Gies, arXiv:1010.xxxx)

- this talk

$$\mathcal{O} \simeq \mu_0^{d_\mathcal{O}} F_\mathcal{O}(N_f) |N_f - N_f^{\rm cr}|^{\frac{d_\mathcal{O}}{|\Theta|}}$$

$$\alpha(m_\tau) = \frac{g^2(m_\tau)}{4\pi} \quad \text{for all } N_f$$

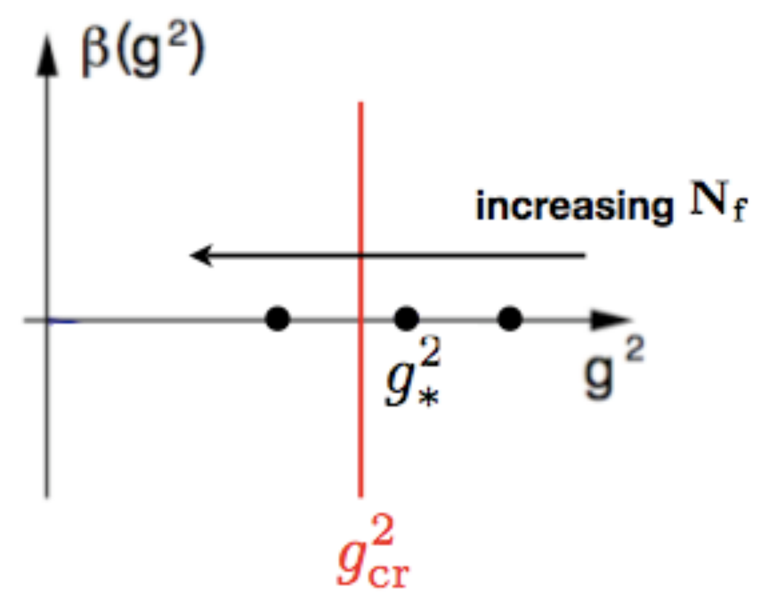


- **Miransky-** or **BKT-type** scaling
(Berezinskii-Kosterlitz-Thouless)

$$\mathcal{O} \simeq \mu_0^{d_\mathcal{O}} \Theta(N_f^{\rm cr} - N_f)\, \mathrm{e}^{-c/|g_*^2(N_f) - g_{\rm cr}^2|^{\frac{1}{2}}}$$

(Miransky ’85, Kosterlitz ’74)

$$\text{consider } \partial_t g^2 = 0 \text{ for all } N_f$$
$$\text{and use } g_*^2(N_f) \text{ as "external" parameter}$$



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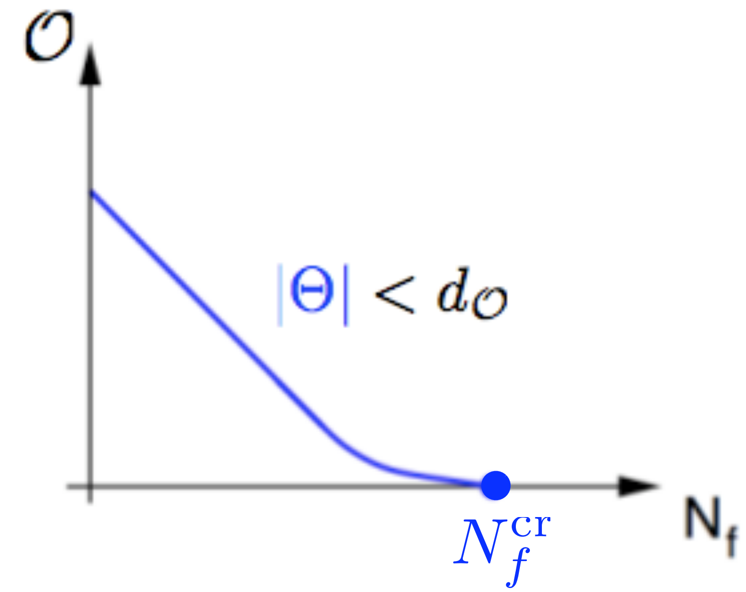

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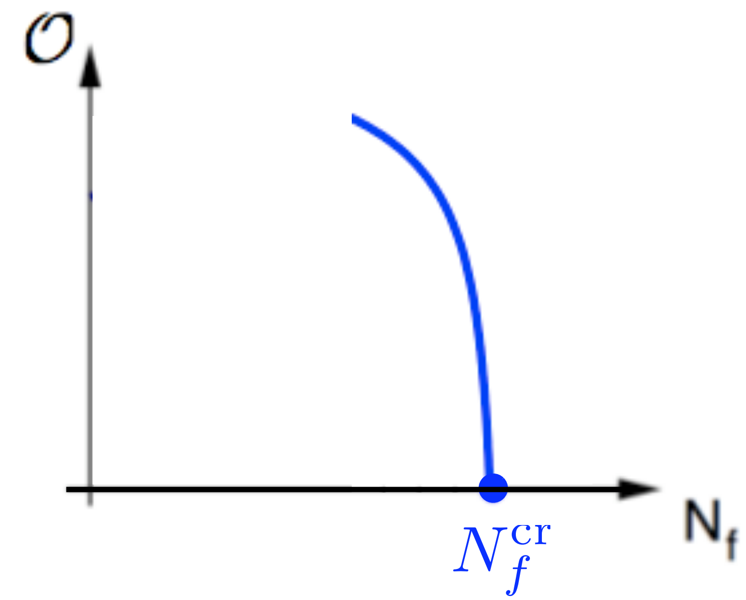


Status around 2003 ...

What is $N_f^{\rm cr}$?

What is $\Theta(N_f^{\rm cr})$?

$$N_f^{\rm cr} = \begin{cases} 12 & \text{(Appelquist et al. '96)} \\ 8 & \text{(Brown et al. '92)} \\ 5 & \text{(Harada \& Yamawaki et al. '00)} \\ 10 & \text{(Kogut et al. '92)} \\ \vdots & \end{cases}$$

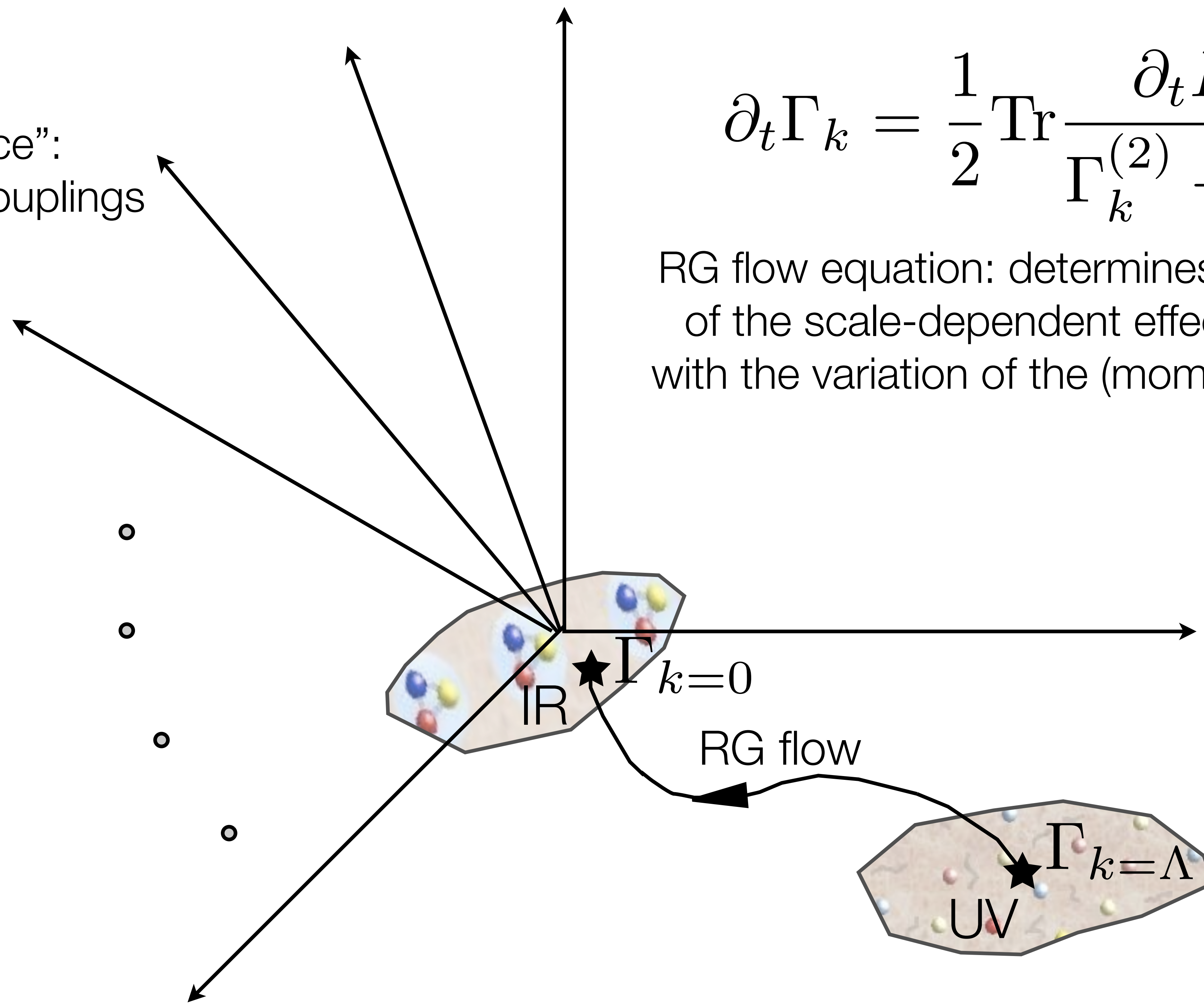
Functional Renormalization Group

(C. Wetterich ‘92)

“Theory space”:
spanned by all couplings

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

RG flow equation: determines the change of the scale-dependent effective action with the variation of the (momentum) scale



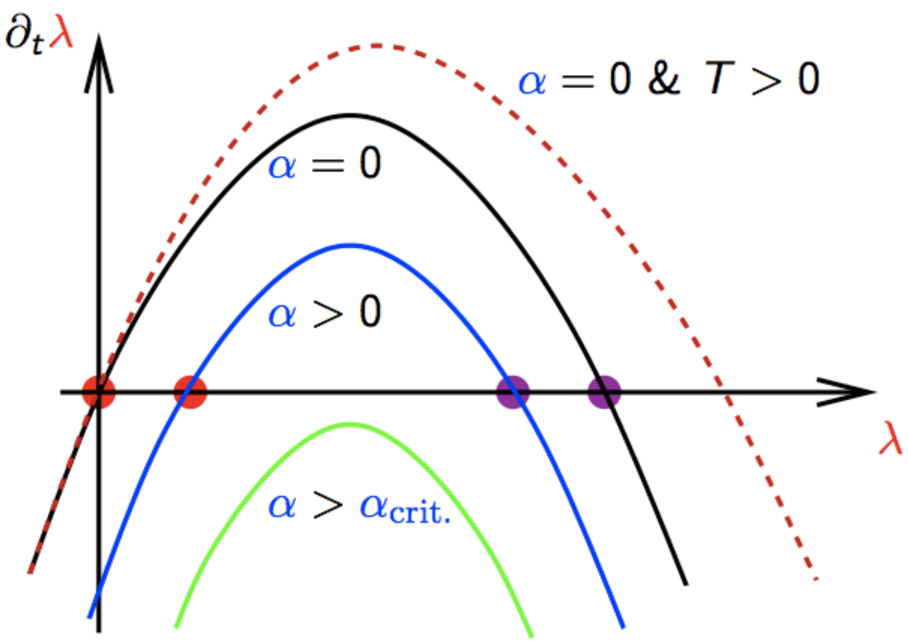
RG flow for the chiral QCD sector

- effective action:

$$\Gamma_k = \int_x \left\{ \frac{\bar{g}^2}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} + w_2 (F^a_{\mu\nu} F^a_{\mu\nu})^2 + w_3 (F^a_{\mu\nu} F^a_{\mu\nu})^3 + \dots \right\}$$
$$+ \int_x \Big\{ \bar{\psi}(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/)\psi + \frac{1}{2}\Big[\frac{\lambda_-}{k^2}(\mathrm{V}-\mathrm{A}) + \frac{\lambda_+}{k^2}(\mathrm{V}+\mathrm{A})$$
$$+ \frac{\lambda_\sigma}{k^2}(\mathrm{S}-\mathrm{P}) + \frac{\lambda_{\mathrm{VA}}}{k^2}[2(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} + (1/N_c)(\mathrm{V}-\mathrm{A})]\Big]\Big\}$$

- no Fierz-ambiguity
- four-fermion interactions generated by quark-gluon dynamics $\left(\lim_{\Lambda\to\infty} \lambda_i = 0 \right)$
- truncation checks: momentum dependencies, regulator dependencies, higher order interactions

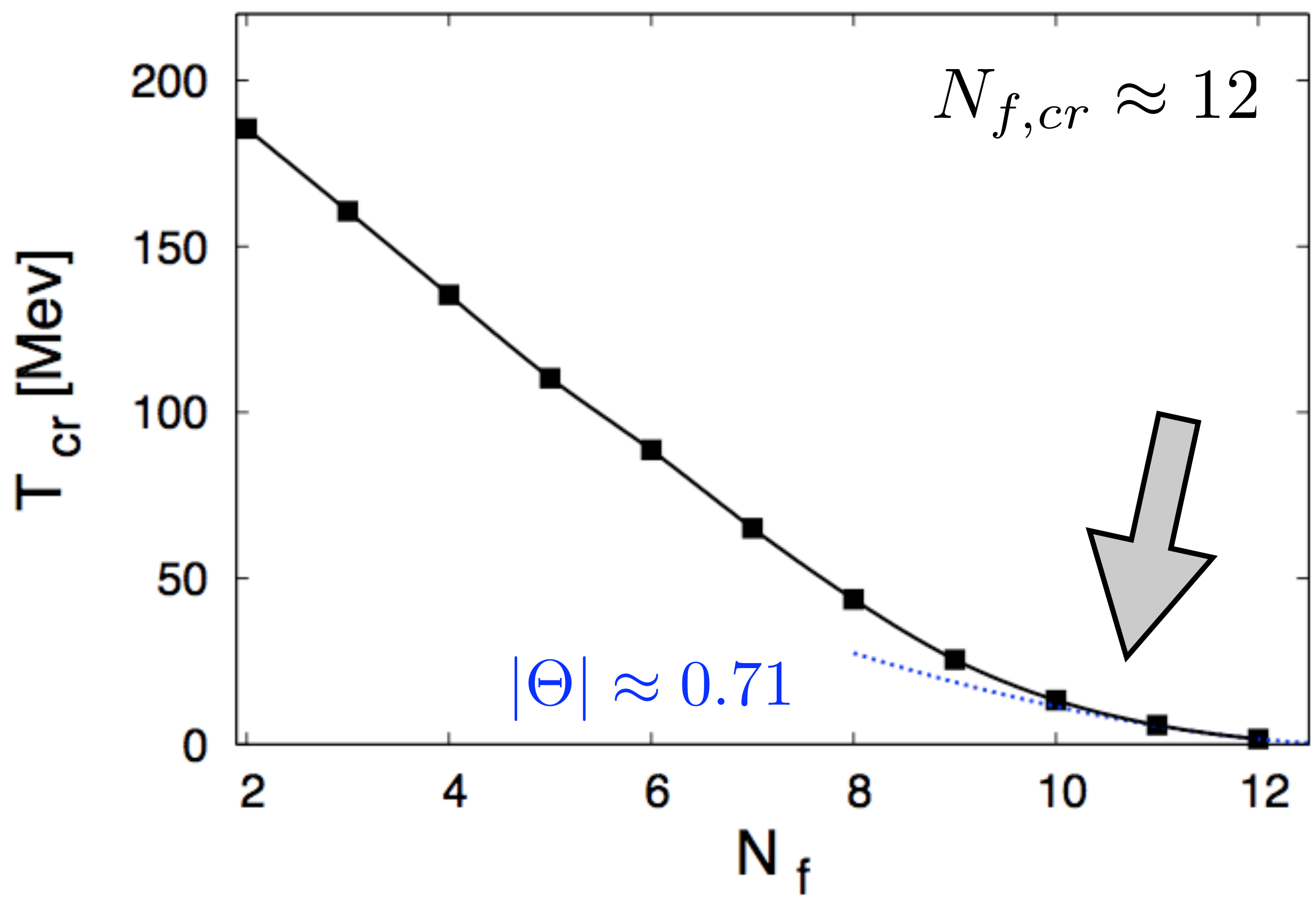
(H. Gies, J. Jaeckel, C. Wetterich '04, H. Gies, C. Wetterich '02, JB '08)



(J. Jaeckel & H. Gies '05; JB, H. Gies '05)

Many-flavor QCD

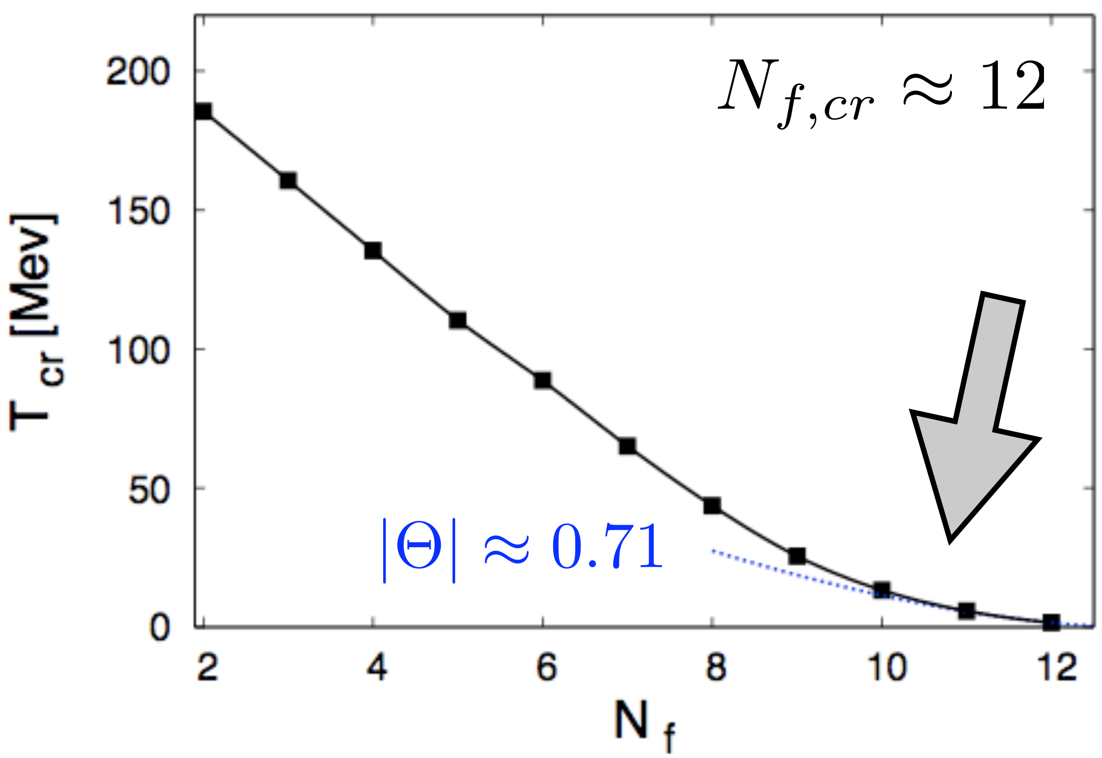
(JB, H. Gies '05,'06, '09)

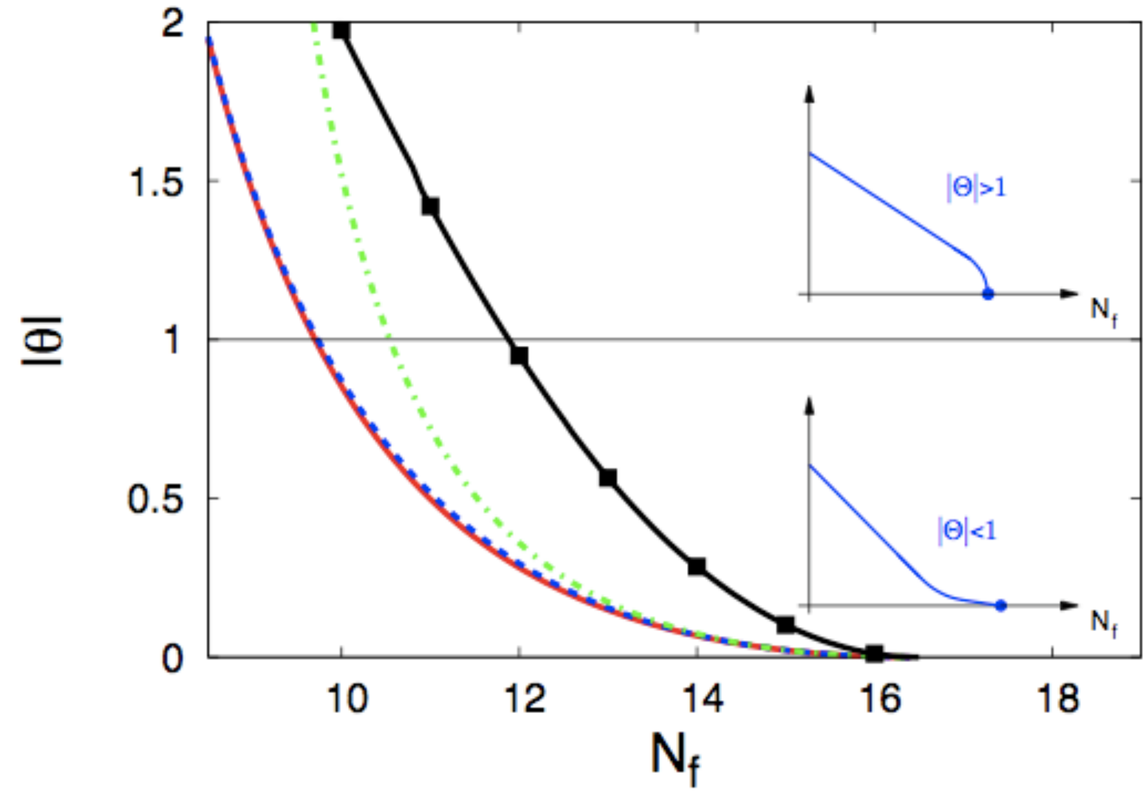


- critical number (RG error estimate): $N_{f,cr} \simeq 10 \,..\, 12$ (H. Gies & J. Jaeckel '05; JB & H. Gies '05)
- walking technicolor: $N_{f,cr} \simeq 12$ (Dietrich, Sannino, Tuominen '05; Dietrich, Sannino '06)
- state-of-the-art lattice studies: $9 < N_{f,cr} \lesssim 12$ (Appelquist, Fleming, Neil '08, '09; Deuzeman, Lombardo, Pallante '08; Fodor et al. '08, '09; Fodor, Holland, Kuti, Nogradi, Schroeder '09; Jin, Mawhinney '09)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

Many-flavor QCD

(JB, H. Gies '05,'06, '09)





- critical number (RG error estimate): $N_{f,cr} \simeq 10\,..\,12$ (H. Gies & J. Jaeckel '05; JB & H. Gies '05)
- walking technicolor: $N_{f,cr} \simeq 12$ (Dietrich, Sannino, Tuominen '05; Dietrich, Sannino '06)
- state-of-the-art lattice studies: $9 < N_{f,cr} \lesssim 12$ (Appelquist, Fleming, Neil '08, '09; Deuzeman, Lombardo, Pallante '08; Fodor et al. '08, '09; Fodor, Holland, Kuti, Nogradi, Schroeder '09; Jin, Mawhinney '09)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

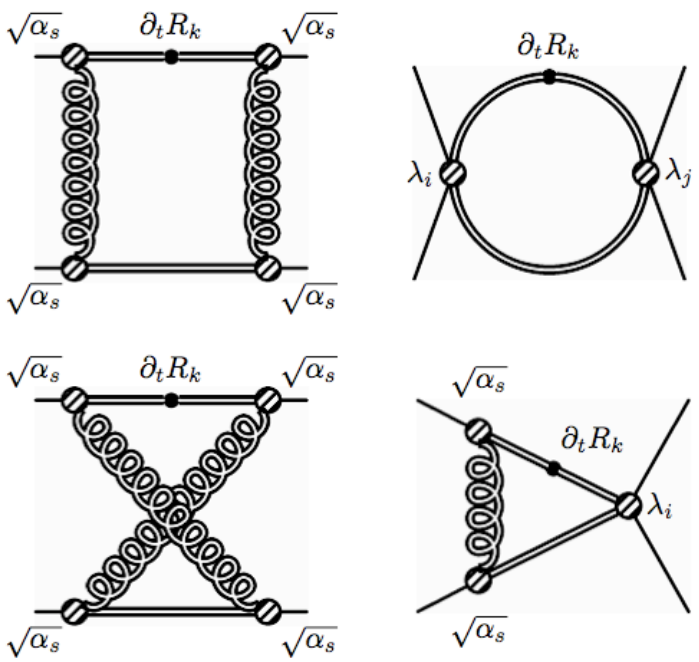
Conclusions

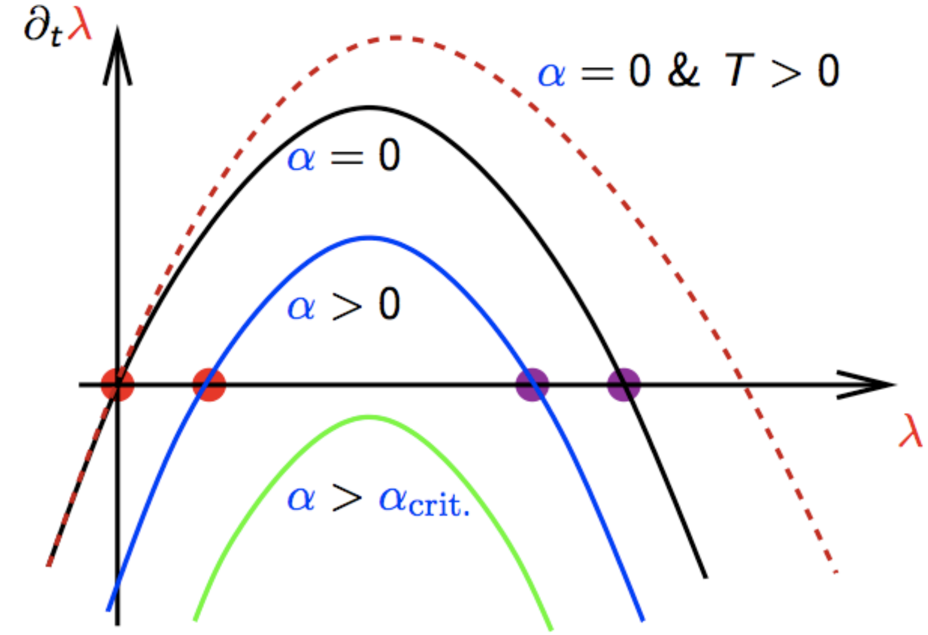
- critical number of quark flavors for SU(3): $N_{f,cr} \approx 10..12$
- scaling of physical observables near $N_{f,cr}$ is determined by the underlying IR fixed point scenario (testable prediction!)

Outlook

- corrections to scaling due to (current) quark mass (and finite volume)
- testing other theories, e. g. QED3 (together with H. Gies, C. Fischer)

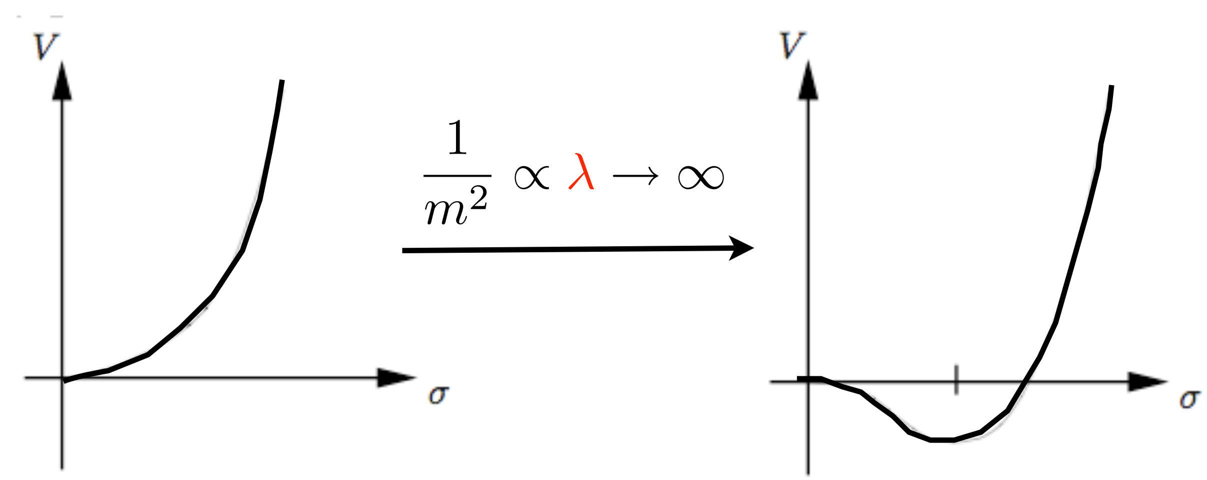
“Criticality” at zero and finite temperature



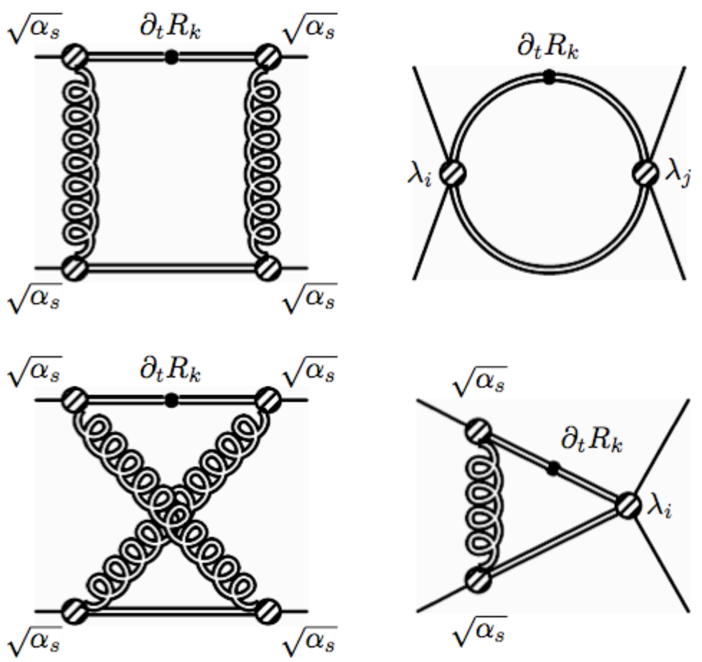


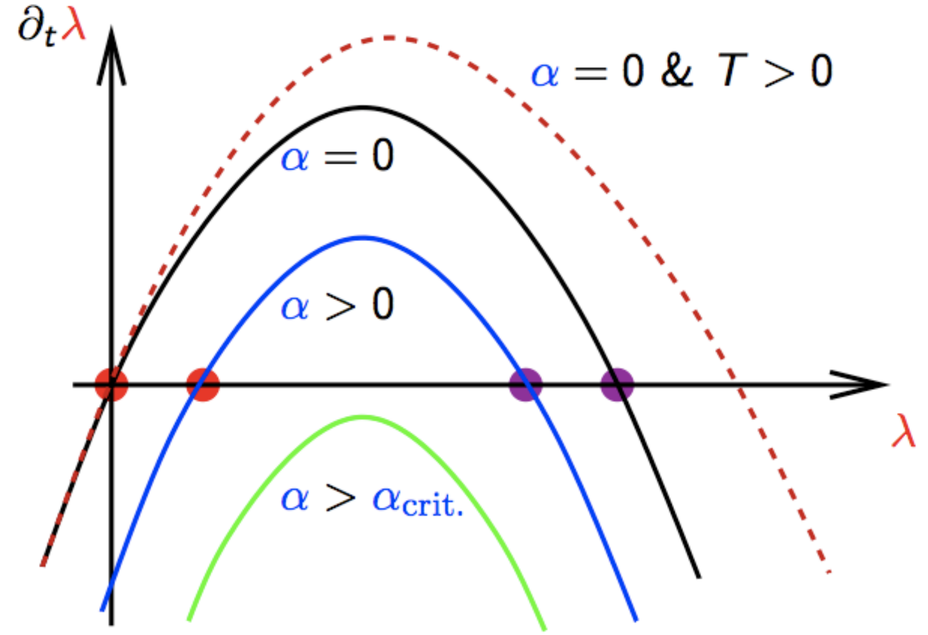
- flow of four-fermion couplings:

$$\partial_t \lambda = 2\lambda - \lambda A(\tfrac{T}{k})\lambda - b(\tfrac{T}{k})\lambda\alpha_s - c(\tfrac{T}{k}){\alpha_s}^2$$



“Criticality” at zero and finite temperature





- critical gauge coupling α_{cr} :

  if $\alpha_s > \alpha_{cr}$ ⟹ no fixed points ⟹ χSB

- at finite temperature: (JB, H. Gies ’05)

$$\alpha_{cr}(T/k) > \alpha_{cr}(T=0)$$

quarks acquire a thermal mass