Early Cosmology From String Theory

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Based on works in collaboration with :

Hagedorn era:	- C. Angelantonj, C. Kounnas, N. Toumbas
	- I. Florakis, C. Kounnas, N. Toumbas
Later evolution:	- T. Catelin-Jullien, C. Kounnas, N. Toumbas
Attractor mechanisms:	- F. Bourliot, J. Estes, C. Kounnas
	- J. Estes, C. Kounnas
Type I cosmology:	- L. Liu, J. Estes (in progress)

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Introduction

- In string theory, many CFT's on the worldsheet describe **flat** or **AdS**-like backgrounds : **Static**.

  ✓ Since the Universe is at finite T, we can deform these CFT's in order to describe Universes filled with a **gas of string** modes : The backreaction will induce a **cosmological evolution**.

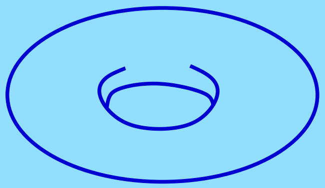
  ✓ Since we want $\Lambda \approx 0$ at late times, we focus on the **flat case**.

- In field theory, the canonical partition function $\mathcal{Z}_{\rm th} = {\rm Tr}\, e^{-\beta H}$ can be evaluated by a path integral in **Euclidean time compactified** on $S^1(R_0)$, where $\beta = 2\,\pi R_0$.

$$F = -\frac{\ln \mathcal{Z}_{\rm th}}{\beta} = \int_0^{+\infty} \frac{dl}{2l}(\cdots)$$

is **UV divergent**

- In string theory, the **infinite number of states** implies a new symmetry (modular invariance) :



$$F = -\frac{\qquad}{\beta} = \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2}(\cdots)$$

[O'Brien, Tan (87)]
[McClain, David, Roth (87)]
[Ditsas, Floratos (88)]

is **UV finite**

- **New problem :** When T increases, the number of string modes that can be thermalized increases exponentially.

  - $\Longrightarrow$ Divergence of $\mathcal{Z}_{\rm th}$ above an **Hagedorn temperature** $T_H = O(M_{\rm string})$.

  - It is believed that this signals a **phase transition** at the maximal temperature T_H .

  - When we go backward in time, this should occur before we reach an initial singularity: **This may be good to resolve the Big Bang**.

- The general picture we find looks like :

Hagedorn Era	Intermediate Era	Standard cosmology:
$T \approx M_{\text{string}}$	$T < M_{\text{string}}$	baryogenesis, nucleosynthesis,...

✓ Propose an approach to describe dynamically the **phase transition** and **avoid the Big Bang singularity.**

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✓ Propose an approach to describe dynamically the **phase transition** and **avoid the Big Bang singularity.**

✓ The **dilaton,** $\boldsymbol{T}$ **and** $\boldsymbol{M}$ (the $\mathcal{N}=1$ susy breaking scale) are attracted to lower values: **Good for hierarchy** $\boldsymbol{M \ll M_{\rm string}}$**.**

✓ The **other moduli** can be **stabilized** and **avoid the «moduli problem».**

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✓ The **dilaton, T and M** (the $\mathcal{N}=1$ susy breaking scale) are attracted to lower values: **Good for hierarchy $M \ll M_{\text{string}}$.**

✓ The **other moduli** can be **stabilized** and **avoid the «moduli problem».**

✓ M and the dilaton need to be stabilized. It is expected to be the case when IR effects enter the game : Electroweak radiative breaking ($T \approx M_{EW}$), gaugino condensation.

Hagedorn Era

[Angelantonj, Kounnas, H.P., Toumbas (09)]

- Euclidean Type II on $S^1(R_0) \times T^{D-1}(R_{\rm box}) \times T^{9-D} \times S^1(R_9)$

$$Z = R_{\rm box}^{D-1} \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^{\frac{D+1}{2}}} \, \frac{1}{2}\sum_{a,b}(-)^{a+b+ab}\theta[^a_b]^4 \, \frac{1}{2}\sum_{\bar a,\bar b}(-)^{\bar a+\bar b+\bar a\bar b}\bar\theta[^{\bar a}_{\bar b}]^4 \, \frac{\Gamma_{(9-D,9-D)}}{(\eta\bar\eta)^{12}}$$

$$\frac{R_0}{\sqrt{\tau_2}}\sum_{n_0,\tilde m_0} e^{-\frac{\pi R_0^2}{\tau_2}|n_0\tau+\tilde m_0|^2} \qquad \frac{R_9}{\sqrt{\tau_2}}\sum_{n_9,\tilde m_9} e^{-\frac{\pi R_9^2}{\tau_2}|n_9\tau+\tilde m_9|^2}$$

$$(-)^{(a+\bar a)\tilde m_0+(b+\bar b)n_0} \qquad (-)^{\bar a\tilde m_9+\bar b n_9+\tilde m_9 n_9}$$

$\Rightarrow$ Finite T $\qquad\qquad$ $\Rightarrow \mathcal{N} = (4{,}4) \to (4{,}0)$

- Alternatively, $(-)^{a\tilde m_0+bn_0+\tilde m_0 n_0}$ breaks $(4{,}0) \to (0{,}0)$: **Still cosmological, but no obvious link with temperature.**

Thermal model : Reversed GSO in the odd n_0, n_9 winding sectors $\Longrightarrow$ $O_8\bar{O}_8$ character,

$$m^2_{O\bar{O}} = R_0^2 - 2 \quad \Longrightarrow \quad R_H = \sqrt{2}$$

✓ For $R_0 > R_H$: Unfold the fundamental domain

$$Z = \int_{\mathcal{F}} d^2\tau \sum_{n_0} (\cdots) \longrightarrow \int_{\text{strip}} d^2\tau\, (\cdots) \quad \text{where} \quad n_0 = 0$$

to bring Z in the form of a canonical partition function :

$$e^Z \equiv \mathcal{Z}_{\text{th}} = \text{Tr}\, e^{-\beta H} \quad \text{where} \quad \beta = 2\pi R_0$$

For the 2[d] model :

$$m^2_{O\bar{O}} = \left(\frac{1}{2R_0} - R_0\right)^2 + \left(\frac{1}{2R_9} - R_9\right)^2 \geq 0$$

✓ The would-be tachyons generate an enhanced $SO(4)_L \times SO(4)_R$ at the fermionic point, $R_0 = R_9 = 1/\sqrt{2}$.

✓ Since $\quad (-)^{a\tilde{m}_0} = (-)^{(a+\bar{a})\tilde{m}_0} \; (-)^{\bar{a}\tilde{m}_0}$

we identify a **dressed canonical partition function** :

$$e^Z \equiv \mathcal{Z}_{\rm th} = {\rm Tr}\, e^{-\beta H}(-)^{\bar{a}}$$

Total Right moving Ramond charge of the muti-particle eigenstates of the Hamiltonian

where
$$\left|\begin{array}{lll} \beta = 2\pi R_0 & \text{for} & R_0 > \dfrac{1}{\sqrt{2}} \\[2ex] \beta = 2\pi \dfrac{1}{2R_0} & \text{for} & R_0 < \dfrac{1}{\sqrt{2}} \qquad \text{(by T-duality)} \end{array}\right.$$

- Note that $$e^Z \equiv \mathcal{Z}_{\rm th} = \mathrm{Tr}\, e^{-\beta H}(-)^{\bar a}$$

differs from the undressed trace for multi-particle states which involve at least one mode with Right moving Ramond charge 1.

✓ They have masses $\geq \dfrac{1}{2R_9}$ or R_9 .

✓ If $R_9 \approx 1$, these masses $= O(M_{\rm string})$.

✓ In the **Intermediate Era**, $T < M_{\rm string}$, the multi-particle states that contain them are Boltzmann suppressed :

$$\Longrightarrow \qquad \mathrm{Tr}\, e^{-\beta H}(-)^{\bar a} \simeq \mathrm{Tr}\, e^{-\beta H}$$

✓ **The two models cannot be distinguished in the Intermediate Era and thus in the present world !**

Other point of view :

- Change of basis in the tachyon free model : $x'_9 = x_9 - x_0$

$$\frac{\sqrt{G'}}{\tau_2}\sum_{\vec{n},\vec{\tilde{m}}} e^{-\frac{\pi}{\tau_2}(G'+B')_{ij}(n\tau+\tilde{m})_i(n\bar{\tau}+\tilde{m})_j} \; (-)^{(a+\bar{a})\tilde{m}_0+(b+\bar{b})n_0+\tilde{m}_0 n_0} \; (-)^{\bar{a}\tilde{m}_9+\bar{b}n_9+\tilde{m}_9 n_9}$$

where $G'_{ij} = \begin{pmatrix} R_0^2+\mu R_9^2 & \mu R_9^2 \\ \mu R_9^2 & R_9^2 \end{pmatrix}$, $B'_{ij} = \begin{pmatrix} 0 & \tilde{\mu} R_9^2 \\ -\tilde{\mu} R_9^2 & 0 \end{pmatrix}$

$$\mu = 1\,,\;\; \tilde{\mu} = 1/2$$

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- They are **Wilson lines** for the $U(1)$'s $G'_{\mu 9}$ and $B'_{\mu 9}$ along the Euclidean $S^1(R_0)$ $\Longrightarrow$ non-dynamical, fixed parameters.

- The **tachyons** found for $\mu = \tilde{\mu} = 0$ **are charged under these $U(1)$'s**.

- **The WL deform the thermal vacuum in order to lift the mass^2 to positive values !**

Simpler Example in $D = 2$

[Florakis, Kounnas, H.P., Toumbas (10)]

- Hybrid A or B: Euclidean IIA or IIB on $S^1(R_0) \times S^1(R_{\text{box}}) \times \mathscr{M}_8$

$$Z = R_{\text{box}} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^{3/2}} \frac{1}{2} \sum_{a,b} (-)^{a+b+\nu ab} \theta[^a_b]^4 \frac{\Gamma_{E_8}}{\eta^{12}} (\bar{V}_{24} - \bar{S}_{24})$$

$$\frac{R_0}{\sqrt{\tau_2}} \sum_{n_0,\tilde{m}_0} e^{-\frac{\pi R_0^2}{\tau_2}|n_0\tau+\tilde{m}_0|^2} \quad (-)^{a\tilde{m}_0+bn_0+\tilde{m}_0 n_0}$$

✓ Right movers satisfy a massive boson/fermion degeneracy symmetry MSDS: [Kounnas (08)] [Florakis, Kounnas (09)][Florakis, Kounnas,Toumbas (10)]

$$\bar{V}_{24} - \bar{S}_{24} = 24 \longleftarrow$$

Unpaired massless modes in NS_{Right}
[See I. Florakis' talk]

✓ $R_2, \ldots, R_9$ are stabilized at $1/\sqrt{2}$: enhanced $SU(2)^8_R$.

$\Longrightarrow$ Conventional canonical ensemble in the Intermediate Era

$$\frac{Z}{V_{\text{box}}} = 24 \begin{cases} \dfrac{1}{R_0} & \text{for } R_0 > 1/\sqrt{2} \\ 2R_0 & \text{for } R_0 < 1/\sqrt{2} \end{cases}$$

$$= 24\left(\frac{1}{2R_0} + R_0\right) - 24\left|\frac{1}{2R_0} - R_0\right|$$

[Similar to non-critical Heterotic string analyzed by Davis, Larsen, Seiberg]

- ✓ Always finite : **No tachyon** for any R_0.
- ✓ Enhanced symmetry $U(1)_L \to SU(2)_L$ at $R_0 = 1/\sqrt{2}$.
- ✓ $\partial_{R_0} Z$ is discontinusous there : **Phase transition**.

In Hybrid B

- Integrated over the fundamental domain, Z involves Left-moving characters V_8, S_8, and O_8, C_8 (due to reversed GSO).

- Unfold the fundamental domain **in the phase** $R_0 > 1/\sqrt{2}$:

$$\left(\Gamma_{m_0}\, V_8 - \Gamma_{m_0+1/2}\, S_8\right)\left(\bar{V}_{24} - \bar{S}_{24}\right)$$

$$\Longrightarrow \quad e^Z \equiv \mathcal{Z}_{\rm th} = {\rm Tr}\, e^{-\beta H}(-)^{\bar{a}} \quad \text{where} \quad \beta = 2\pi R_0$$

- Unfold the fundamental domain **in the phase** $R_0 < 1/\sqrt{2}$:

$$\left(\Gamma_{\tilde{n}_0}\, V_8 - \Gamma_{\tilde{n}_0+1/2}\, C_8\right)\left(\bar{V}_{24} - \bar{S}_{24}\right)$$

which is the result for Hybrid A for the T-dual radius

$$\Longrightarrow \quad e^Z \equiv \mathcal{Z}_{\rm th} = {\rm Tr}\, e^{-\beta H}(-)^{\bar{a}} \quad \text{where} \quad \beta = 2\pi\, \frac{1}{2R_0}$$

The phase transition at the fermionic point changes :

Hybrid B with KK Matsubara modes for $R_0 > 1/\sqrt{2}$ → Hybrid A with winding Matsubara modes for $R_0 < 1/\sqrt{2}$

$p_L = p_R$ along $S^1(R_0)$

$$p_L = 1/2 \quad , \; p_R = 1/2$$

$$p_L = -1 \quad , \; p_R = 0$$

Additional massless states

$$p_L = -1/2 \; , \; p_R = 1/2$$

$p_L =$ -p_R

✓ With marginal operators : $O_- \left| S_8; \frac{1}{2}, \frac{1}{2} \right\rangle \to \left| C_8; -\frac{1}{2}, \frac{1}{2} \right\rangle$

- In both phases, the «deformed free energy density» (for the dressed Tr) is $$\frac{F}{V_{\rm box}} \equiv -\frac{Z}{\beta V_{\rm box}} = -\frac{\kappa}{\beta^2} \quad \text{where} \quad \kappa = 48\pi$$

  - ✓ It is that of a standard radiation, even in the Hagedorn Era $R_0 \simeq 1/\sqrt{2}$: **Why ?**

  - ✓ For a gas of a single Bosonic (or Fermionic) degree of freedom, with Right moving Ramond charge $\bar{a}$,

$$\ln \, \mathrm{Tr}\, e^{-\beta H}(-)^{\bar{a}} = \mp \sum_k \ln\left(1 \mp (-)^{\bar{a}} e^{-\beta\omega_k}\right)$$

$$\Longrightarrow \qquad (\text{Boson}, \bar{a}) \text{ and } (\text{Fermion}, \bar{a}+1) \text{ have opposite } F.$$

  - ✓ But by definition, the MSDS symmetry on the Right matches them when they are massive ! We are left with a **standard free energy** for thermal **radiation** of bosons and fermions.

- This allows us to **describe the cosmological phase transition** that occurs at $R_0 = 1/\sqrt{2}$.

  - ✓ The universe is flat Minkowski we started with, on which the source F we have computed at 1-loop backreacts.

  - ✓ Convention : $t = 0$ when $R_0 = 1/\sqrt{2}$.

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- For $\boldsymbol{t > 0}$, we know the tree level + 1-loop effective action, exact in α' at the 2-derivatives level :

$$\int_{t>0} dtdx\, \underbrace{\beta a}_{\boxed{2\pi R_0}} \left[e^{-2\phi}\left(\frac{R}{2} + 2(\partial\phi)^2\right) - \frac{\kappa}{\beta^2}\right]$$

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  - ✓ For $\boldsymbol{t > 0}$, we know the tree level + 1-loop effective action, exact in α' at the 2-derivatives level :

  - ✓ Also for $\boldsymbol{t < 0}$:

$$\int_{t<0} dtdx\, \underbrace{\beta a}_{\boxed{2\pi\frac{1}{2R_0}}}\Big[\cdots\Big] + \int_{t>0} dtdx\, \underbrace{\beta a}_{\boxed{2\pi R_0}}\left[e^{-2\phi}\left(\frac{R}{2} + 2(\partial\phi)^2\right) - \frac{\kappa}{\beta^2}\right]$$

Convention : laps function $\equiv \beta$

- **At $t = 0$**, the thermal system contains additional massless states, which trigger the phase transition, Hybrid A→B.

- They are 24 complex scalars χ_i, with $n_0 = m_0 = \pm 1$. They do not exist for $t > 0$ (or $t < 0$), where the thermal system contains pure KK (or winding) Matsubara modes only.

- **Their tree level effective action is thus in space only :**

$$\int dx\, \sqrt{g_{11}}\, e^{-2\phi} \left(-\, g^{11}\, \frac{d\chi_i}{dx} \frac{d\bar{\chi}_i}{dx} \right) \quad \Longrightarrow \quad \chi_i = \alpha_i + \gamma_i \sqrt{g_{11}}\, x$$

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- In total, the action is :

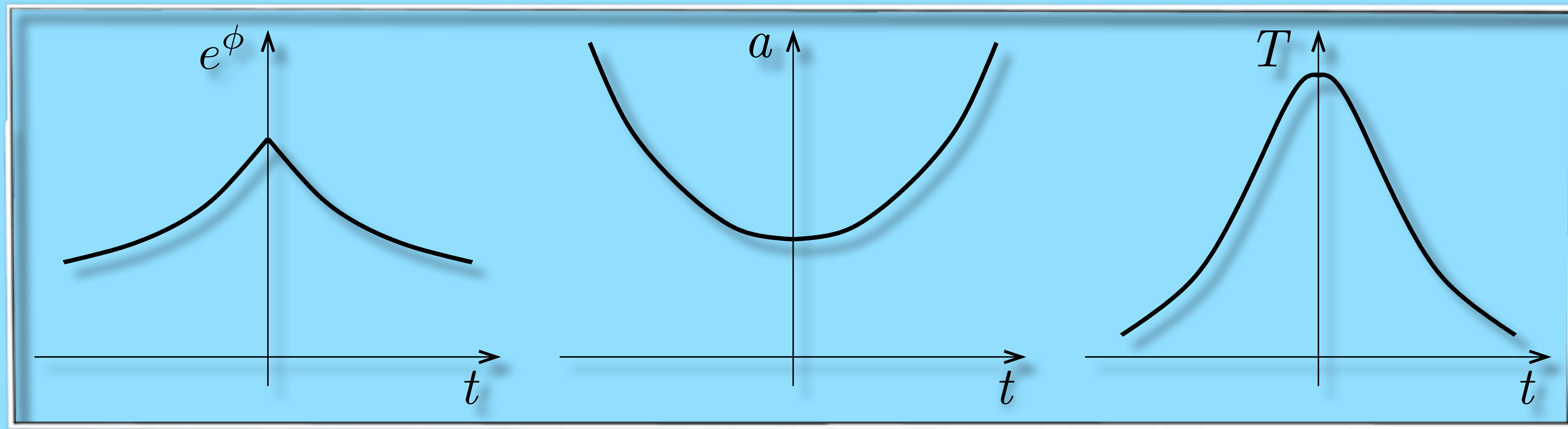
$$\int dt dx\, \beta a \left[e^{-2\phi} \left(\frac{R}{2} + 2(\partial\phi)^2 \right) - \frac{\kappa}{\beta^2} \right] - \int dt dx\, a\, e^{-2\phi}\, \delta(t) \overbrace{\sum_i |\gamma_i|^2}^{\text{Tension of a brane-like object}}$$

- The constant gradients γ_i at every point in space introduce some non-trivial winding quantum number in the pure KK_0 thermal vacuum at the transition. This is analogous to the condensation of winding tachyons.

Solution :

$$e^{\phi} = \frac{e^{\phi_0}}{\sqrt{1+2C\,|t|}}\ , \quad a = \frac{a_0}{\sqrt{1+2C\,|t|}}\, e^{C|t|}\ , \quad T = T_c\, e^{-C|t|}\, \sqrt{1+2C\,|t|}$$

$$\text{where} \qquad C = e^{\phi_0}\sqrt{2\kappa} \qquad \text{and} \qquad T_c = \frac{\sqrt{2}}{2\pi}\ .$$



- Bouncing cosmology : **No Big Bang singularity.**
- Fully **perturbative.**
- Phase transition at the **maximal temperature** T_c, where there is a **conical singularity in** $\phi(t)$ only and a **constant entropy.**

Intermediate Era, $T < M_{\text{string}}$

- **In realistic models :**
  - ✓ All moduli should get a mass (to not modify Newton's law).
  - ✓ $\mathcal{N}=1$ susy should be softly broken at low energy (to solve the hierarchy problem) :

$$M \simeq 1 \text{ TeV} \quad \Longrightarrow \quad \text{moduli masses } M \simeq 1 \text{ TeV} .$$

[Coughlan, Fishler, Kolb, Raby, Ross (83)]
[Coughlan, Holman, Ramond, Ross (84)]
[Goncharov, Linde, Vysotsky (84)]

- **Moduli problem :**
  - ✓ When massive, the moduli oscillate around their minima :

$$\Longrightarrow \quad \rho_{\text{moduli}} \propto T^3 \;\; \text{will dominate} \;\; \rho_{\text{rad}} \propto T^4 .$$

  - ✓ If stable : overclose the Universe.
  - ✓ If decay into radiation : alter baryogenesis.

- We would like first to ask how to obtain $M \ll M_{\text{string}}$?

  - If in the **Hagedorn Era** we now have $\boldsymbol{T \approx M \approx M_{\text{string}}}$, we are going to see that in the following **Intermediate Era**, both **$T(t)$ and $M(t)$ are attracted down to M_{EW}, where $M(t)$ is supposed to be stabilized.**

  - The moduli have **decreasing mass $M(t)$.** This is an intermediate situation, between a constant mass and a vanishing mass !

  - The energy stored in their oscillations is dominated by thermal energy ! **There is no moduli problem.**

[Bourliot, Kounnas, H.P (09)]
[Bourliot, Estes, Kounnas, H.P. (09)]
[Estes, Kounnas, H.P. (10)]

$\mathcal{N}=1$ Heterotic orbifold in $D=4$ at finite $T < M_{\text{string}}$

- If the internal radii satisfy $\dfrac{1}{R_0} < R_i < R_0$, the masses

$$\left.\begin{array}{r} M_{\text{string}}\ \text{(for oscillators)} \\ \dfrac{1}{R_i}\ \text{(for KK states)} \\ R_i\ \text{(for winding states)} \end{array}\right\} > \frac{1}{R_0} \propto T$$

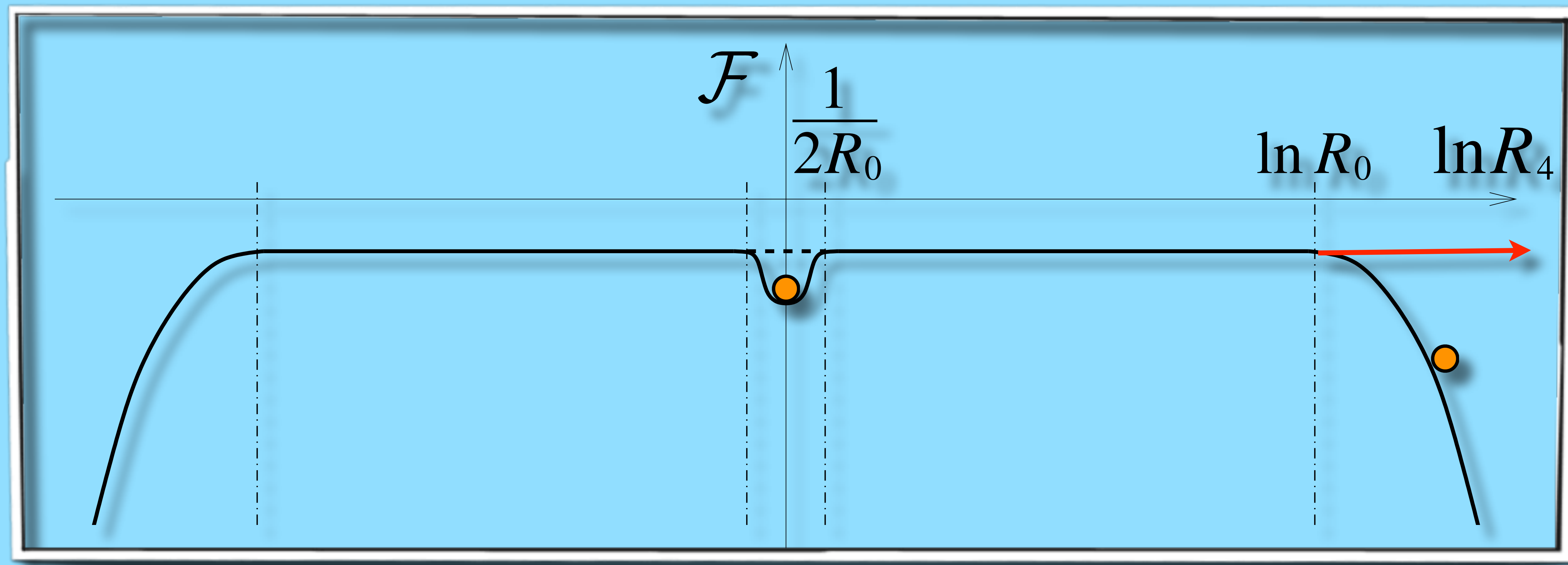
Matsubara excitations (KK_0) of n massless states

Boltzmann supression

$$\mathcal{F} = \frac{F}{V_{\text{box}}} = -T^4\, n \sum_{\tilde{k}_0} \frac{1}{\pi^2(2\tilde{k}_0+1)^4} + \mathcal{O}(e^{-\frac{\text{Mass}}{T}})$$

- In fact, this «constant» can vary in the neighborhood of enhanced symmetry points : $n \to n + n_{enhan}$.

✓ E.g., $\mathcal{F}$ is the effective potential at finite T for R_4 :



✓ **$R_4(t)$ can be stabilized at 1,** which is the **$SU(2)$ enhanced symmetry point.**

✓ **This occurs with a mass $T(t)$, which drops.**

✓ NB: If $R_4(t) > R_0(t)$, it is always caught by the increasing size of the plateau. Along the plateau, R_4 freezes or is stabilized $\Rightarrow$ We do not decompactify to 10 dimensions !

We add $\mathcal{N}=1$ susy breaking : $T \propto \frac{1}{R_0}$, $M \propto \frac{1}{R_9}$

- The light states are the KK_0 (Matsubara) modes and the KK_9 modes :

$$\mathcal{F} = -T^4\, n\, f\Big(\frac{M}{T}\Big) + \mathcal{O}(e^{-\frac{\text{Mass}}{T}}, e^{-\frac{\text{Mass}}{M}})$$

which in fact is $\mathcal{F} = -T^4\, g\Big(\frac{M}{T}, R_4\Big) + \mathcal{O}(e^{-\frac{\text{Mass}}{T}}, e^{-\frac{\text{Mass}}{M}})$

to interpolate the jump in n at the enhanced sym point $R_4 = 1$.

- E.o.m for $e^z = M/T$,

$$\boxed{g + \partial_z g}$$

$$(\cdots)\ddot{z} + (\cdots)\dot{z} + \frac{\partial \mathcal{V}}{\partial z} = 0$$

- **When $\mathcal{V}$ admits a minimum z_c, there is attraction to a particular solution**

$$e^{z_c}\, T(t) = M(t) \propto \frac{1}{a(t)} \propto e^{4\phi(t)} \propto \frac{1}{\sqrt{t}} \quad \text{where} \quad H^2 \propto T^4 \ , \quad R_4(t) \equiv 1$$

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is **radiation-like :** $\Big[\rho_{\rm th} + \text{kinetic}\Big] = \underbrace{\gamma\left(\frac{M}{T}, R_4\right)}_{3} \cdot \Big[P_{\rm th} + \text{kinetic}\Big]$

$\rho_{\rm th} = \overbrace{\gamma_{\rm th}\left(\frac{M}{T}, R_4\right)}^{4} \cdot P_{\rm th} \quad \Longrightarrow \quad$ this solution has **nothing to do with the usual Radiation Era** after EW breaking.

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- ✓ $\rho_{\rm th} = \overbrace{\gamma_{\rm th}\left(\frac{M}{T}, R_4\right)}^{4} \cdot P_{\rm th} \quad \Longrightarrow$ this solution has **nothing to do with the usual Radiation Era** after EW breaking.

- ✓ The kinetic energy of the oscillations of R_4 and z in their potentials $\mathcal{F}$ and $\mathcal{V}$ is negligible in Friedmann's equation : **No moduli problem**.

- ✓ Since **the gravitino (and modulini)** have mass $M(t)$, which drops with $T(t)$, they are **never thermally produced abundantly.** This is not the case in models where their masses are supposed to be constant = 1 TeV : **gravitino problem.**

✓ We need to know when the potential $\mathcal{V}$ for $z = \ln(M/T)$ admits a minimum z_c : [Catelin-Jullien, Kounnas, H.P, Toumbas (07)]

$$\boxed{\begin{aligned} Z \;=\; & \;\; T^4 \Big(\text{thermal contribution present when } M{=}0\Big) \\ & + M^4 \Big(\text{effective potential contribution present when } T{=}0\Big) \end{aligned}}$$

$> \mathbf{0}$

$> \mathbf{0}$ **or** $\leqslant \mathbf{0}$

✓ It is when the effective potential part is negative that there is balancing and we have a minimum z_c .

Intermediate Era in Type I

[Liu, Estes, H.P (in progress)]

- **Problem 1 :** We have RR moduli.

  Problem 2 : There is no enhanced symmetry point !

- The Heterotic and Type I strings are dual (S-dual in D=10).

  ✓ Their respective gases at finite T must be dual.

  ✓ Since the backreactions we study are **quasi-static**, we can use **Heterotic / Type I duality to derive the Type I cosmology at finite T.**

- Heterotic string compactified on a torus, with $R_i \approx 1$:

$n_i = m_i = \pm 1$

$$\mathcal{F} = -T^D \left\{ n\,\sigma_D + \sum_{i=D}^{9} n_{SU(2)}\, g\left(2\pi R_0 \left| R_i - \frac{1}{R_i} \right| \right) + \cdots \right\}$$

$$\text{where} \quad g(x) = 2 \sum_{\tilde{k}_0} \left(\frac{x}{2\pi |2\tilde{k}_0 + 1|} \right)^{\frac{D}{2}} K_{\frac{D}{2}}\left(x\, |2\tilde{k}_0 + 1| \right)$$

$$2\pi \frac{R_0^I}{\sqrt{\lambda_I}} \left| \frac{R_i^I}{\sqrt{\lambda_I}} - \frac{\sqrt{\lambda_I}}{R_i^I} \right| \quad = \quad 2\pi R_0^I \left| \frac{R_i^I}{\lambda_I} - \frac{1}{R_i^I} \right|$$

Type I coupling in 10 dim, $\lambda_I \gg 1$

D-strings wrapped on $S^1(R_i^I)$ with momentum

$$\Longrightarrow \quad R_i^I \ \text{ is stabilized at } \ \sqrt{\lambda_I}\ .$$

- At late time, $R_i(t) \to 1$ and $\phi(t) \to \phi_0 < 0$

perturbative heterotic string coupling in D dim

$$\Longrightarrow \quad \phi_I(t) \to -\frac{D-6}{4}\,\phi_0 - \text{cst.}$$

✓ **For $D > 6$:** It is not a surprise for solitons to contribute, for a thermal gas at strong coupling.

✓ **For $D \leqslant 6$:** We are at **weak coupling** but **massless solitons are still essential in Type I cosmology.**

- Also **essential in Type I phenomenology**, since there is a **solitonic enhanced gauge symmetry,** e.g. $SU(2)^{10\text{-}D}$.

- Is it awkward ? The contribution to the free energy of these massless solitons can be seen to be equal to that of **E1-instantons** computed by heterotic / Type duality. However, they **span the directions 0 (Euclidean) and i.**

- **Generalization :** In Type I string, with $\mathcal{N}=4$ in D=4, all kinds of moduli can be stabilized this way, except the dilaton :

  - ✓ **RR** 2-form moduli : C_{ij} (dual to B_{ij} in heterotic)
  - ✓ **NS-NS** moduli : G_{ij}
  - ✓ **open string Wilson lines** : Y_j^a