# A New Perspective on QFT from the ERG

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# Outline of this Lecture

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# Textbook renormalization

- Choose an action e.g.

$$S[\phi] = \int d^d x \left[ \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) e^{-S[\phi]}$$

- Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

- If $\delta S$ has the same form as $S$, the theory is renormalizable

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- Don't try to integrate over all fluctuations at once!
- Partition up the modes by introducing an effective scale
- Integrate out degrees of freedom between $\Lambda_0$ and $\Lambda$
- The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

- Demanding invariance of the partition function gives an ERGE

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \ldots$$

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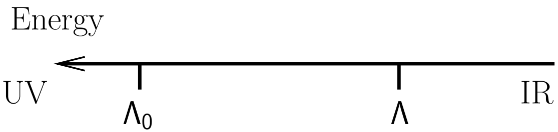


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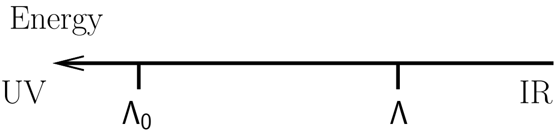


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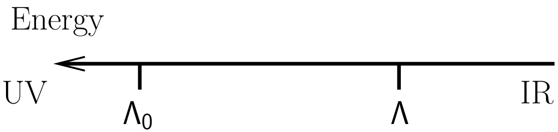


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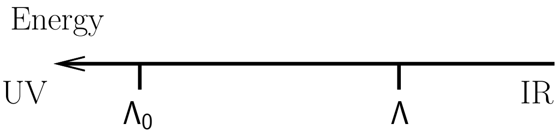


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# What have we gained?

- Suppose we work with dimensionless variables:

$$x \to x/\Lambda, \qquad \phi(x) \to \phi(x)\Lambda^{(d-2)/2}\sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

- Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

Nonperturbatively renormalizable solutions follow from fixed-points

- Either directly: $\partial_t S_\star[\varphi] = 0$
- Or from relevant (source-dependent) perturbations

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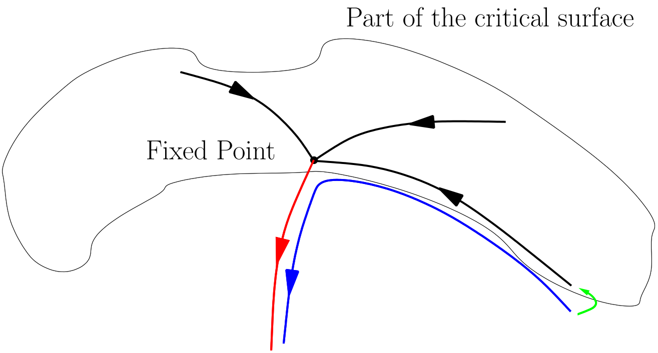

# Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

**Question: What is the link?**

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

**The Simple Answer**

- Suppose that $S_\Lambda$ obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda\to 0} S_\Lambda$

**My aims in this talk**

- To show that there is more to the answer
- To convince you that the question is profound

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# General ERGs

## Formulation

- Demand invariance of the partition function under blocking
- $\Lambda\partial_\Lambda Z = \Lambda\partial_\Lambda \int \mathcal{D}\phi\, e^{-S^{\text{tot}}[\phi]} = \int \mathcal{D}\phi \frac{\delta}{\delta\phi}$
- Parametrizes the blocking procedure

## Alternative point of view

- $\Psi$ corresponds to a field redefinition
- Under RG step $\Lambda \to \Lambda - \delta\Lambda$, $\phi \to$

## Flow Equation

$$-\Lambda\partial_\Lambda S^{\text{tot}} = \int d^d x\, \frac{\delta S^{\text{tot}}}{\delta\phi(x)}\Psi(x) - \int d^d x\, \frac{\delta \Psi(x)}{\delta\phi(x)}$$

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# The Modified Polchinski Equation I

- Introduce UV cutoff function, $K(p^2/\Lambda^2)$
  - quasi-local, with $K(0) = 1$ and $K(\infty) = 0$
- Define regularized propagator $C_\Lambda(p^2) = K(p^2/\Lambda^2)/p^2$
- Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2}\int_p \phi(p) C_\Lambda^{-1}(p^2)\phi(-p)$$

- Make the split

$$S^{\mathrm{tot}} = \hat{S} + S$$

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# Rescalings

- Remove the *canonical* dimensions

$$\tilde{p} = p/\Lambda, \qquad \varphi(\tilde{p}) = \phi(p)\Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

- Choose

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda\frac{d\ln Z}{d\Lambda}$$

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- It is possible to construct a gauge invariant cutoff, using
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  - Pauli-Villars fields
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- No gauge fixing is required at any stage!
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- We must consider expectation values of manifestly gauge invariant operators
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## The Standard Correlation Functions

- Introduce a source term in the bare action

$$Z[J] = \int D\phi \, e^{-S_\Lambda[\phi] + J \cdot \phi}$$

- Extract the connected correlation functions from $W[J] = \ln Z$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \bigg|_{J=0}$$

## Composite Operators

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# New ERG Approach

- Introduce an external field, $J$, with undetermined scaling dimension, $d_J$
- Allow for $J$-dependence of the action

$$S_\Lambda[\phi] \to T_\Lambda[\phi, J]$$

- The flow equation follows as before

$$-\Lambda\partial_\Lambda e^{-T^{\mathrm{tot}}_\Lambda[\phi,J]} = \int d^dx \frac{\delta}{\delta\phi(x)}\left\{\Psi(x)e^{-T^{\mathrm{tot}}_\Lambda[\phi,J]}\right\}$$

- A sensible boundary condition would be

$$\lim_{\Lambda\to\Lambda_0} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J\cdot\phi$$

- But we will not implement the bc in this way

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# Rescalings again

- Treat $\phi$ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p)\Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

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$$\left(\partial_t - D - D^J\right) T_t[\varphi, J] = \frac{\delta T}{\delta \varphi} \cdot K' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot K' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

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Nonperturbatively renormalizable solutions follow from fixed-points

- Either directly: $\partial_t T_\star[\varphi, j] = 0$
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# A Source-Dependent Fixed-Point

- Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

- Then there is always a source-dependent f-p

$$T_\star[\varphi, j] = S_\star[\varphi] + \left[e^{-\bar{j}\cdot\varrho\cdot\delta/\delta\varphi} - 1\right]\left[S_\star[\varphi] + \frac{1}{2}\varphi\cdot C^{-1}(1+\varrho^{-1})\cdot\varphi\right]$$

  - $\bar{j}(p) = j(p)/p^2$
  - $\varrho = \varrho(p^2) = p^2 + O(p^4)$ (quasi-local)

**Two crucial points**

- The solution only works if $d_J = (d + 2 - \eta_\star)/2$
- In dimensionful variables

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# And more. . .

- For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

- Every eigenperturbation, $\mathcal{O}_i$ has a source-dependent extension

$$\tilde{\mathcal{O}}_i[\varphi, j] = e^{\bar{j} \cdot \varrho \cdot \delta/\delta\varphi} \mathcal{O}_i$$

- At the linear level

$$T_t[\varphi, j] = T_\star[\varphi, j] + \sum_i \alpha_i e^{\tilde{\lambda}_i t} \tilde{\mathcal{O}}_i[\varphi, j]$$

where $\tilde{\lambda}_i = \lambda_i$

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- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J \cdot \phi$$

**Conclusion**

If we use the modified Polchinski equation with $\Psi = -\eta\phi/2$

Renormalizability of $S_\Lambda$ implies renormalizability of the standard correlation functions

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# A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
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- Allow arbitrary source dependence
- Search for fixed-point solutions
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**Philosophy**

- The Wilsonian effective action is fundamental
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# Questions

### Modified Polchinski Equation $\psi = -\eta\varphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
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### Other flow equations

- What happens for other flow equations?
- What does this imply for gauge theories?

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*Ask not what quantum field theory can compute for you, but what you can compute for quantum field theory*

# Thank you for listening