◆□▶ ◆□▶ ◆ □▶ ◆ □▶ ○ □ ○ ○ ○ ○

A New Perspective on QFT from the ERG arXiv:1003.1366 [hep-th]

Oliver J. Rosten

Sussex U.

September 2010

Outline of this Lecture



2 ERGEs

Introducing a Source

Outline of this Lecture









Outline of this Lecture









Textbook renormalization

• Choose an action *e.g.*

$$S[\phi] = \int d^d x \left[\frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = rac{1}{\mathcal{Z}} \int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) e^{-S[\phi]}$$

• Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

• If δS has the same form as S, the theory is renormalizable

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

Textbook renormalization

• Choose an action *e.g.*

$$S[\phi] = \int d^{d}x \left[\frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi + \frac{1}{2} m^{2} \phi^{2} + \frac{\lambda}{4!} \phi^{4} \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)e^{-\mathcal{S}[\phi]}$$

• Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

• If δS has the same form as S, the theory is renormalizable

Textbook renormalization

• Choose an action *e.g.*

$$S[\phi] = \int d^d x \left[rac{1}{2} \partial_\mu \phi \partial_\mu \phi + rac{1}{2} m^2 \phi^2 + rac{\lambda}{4!} \phi^4
ight]$$

Choose a UV regulator

• Start computing the correlation functions

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) e^{-S[\phi]}$$

• Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

• If δS has the same form as S, the theory is renormalizable

▲ロト ▲御 ▶ ▲ 臣 ▶ ▲ 臣 ▶ ─ 臣 ─ のへで

Textbook renormalization

• Choose an action *e.g.*

$$S[\phi] = \int d^d x \left[rac{1}{2} \partial_\mu \phi \partial_\mu \phi + rac{1}{2} m^2 \phi^2 + rac{\lambda}{4!} \phi^4
ight]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)e^{-S[\phi]}$$

• Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

• If δS has the same form as S, the theory is renormalizable

▲ロト ▲御 ▶ ▲ 臣 ▶ ▲ 臣 ▶ ─ 臣 ─ のへで

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

Textbook renormalization

• Choose an action *e.g.*

$$S[\phi] = \int d^{d}x \left[\frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi + \frac{1}{2} m^{2} \phi^{2} + \frac{\lambda}{4!} \phi^{4} \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)e^{-S[\phi]}$$

• Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

• If δS has the same form as S, the theory is renormalizable

Textbook renormalization

• Choose an action *e.g.*

$$S[\phi] = \int d^{d}x \left[\frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi + \frac{1}{2} m^{2} \phi^{2} + \frac{\lambda}{4!} \phi^{4} \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)e^{-S[\phi]}$$

• Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

• If δS has the same form as S, the theory is renormalizable

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● の < @

Drawbacks

- In practice, this scheme is perturbative
- It offers no physical intuition

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● の < @

Drawbacks

• In practice, this scheme is perturbative

• It offers no physical intuition

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● の < @

Drawbacks

- In practice, this scheme is perturbative
- It offers no physical intuition

< □ > < □ > < □ > < □ > < □ > < □ > ... □

Wilsonian Renormalization

Don't try to integrate over all fluctuations at once!Partition up the modes by introducing an effective scale

- Integrate out degrees of freedom between Λ_0 and Λ
- The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

Wilsonian Renormalization

• Don't try to integrate over all fluctuations at once!

• Partition up the modes by introducing an effective scale

- \bullet Integrate out degrees of freedom between Λ_0 and Λ
- The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

Wilsonian Renormalization

- Don't try to integrate over all fluctuations at once!
- Partition up the modes by introducing an effective scale

- \bullet Integrate out degrees of freedom between Λ_0 and Λ
- The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

Wilsonian Renormalization

- Don't try to integrate over all fluctuations at once!
- Partition up the modes by introducing an effective scale



 \bullet Integrate out degrees of freedom between Λ_0 and Λ

• The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

Wilsonian Renormalization

- Don't try to integrate over all fluctuations at once!
- Partition up the modes by introducing an effective scale



 \bullet Integrate out degrees of freedom between Λ_0 and Λ

• The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

Wilsonian Renormalization

- Don't try to integrate over all fluctuations at once!
- Partition up the modes by introducing an effective scale

Energy
UV
$$\checkmark$$
 I I IR

- \bullet Integrate out degrees of freedom between Λ_0 and Λ
- The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

Wilsonian Renormalization

- Don't try to integrate over all fluctuations at once!
- Partition up the modes by introducing an effective scale

Energy
UV
$$\checkmark$$
 I I IR

- \bullet Integrate out degrees of freedom between Λ_0 and Λ
- The bare action evolves into the Wilsonian effective action

$$S_{\Lambda_0}[\phi] \to S_{\Lambda}[\phi]$$

$$\Lambda \partial_{\Lambda} S_{\Lambda}[\phi] = \dots$$

What have we gained?

Suppose we work with dimensionless variables:

$$x \to x/\Lambda, \qquad \phi(x) \to \phi(x)\Lambda^{(d-2)/2}\sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

- Either directly: $\partial_t S_{\star}[\varphi] = 0$
- Or from relevant (source-dependent) perturbations

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

What have we gained?

• Suppose we work with dimensionless variables:

$$x o x/\Lambda, \qquad \phi(x) o \phi(x) \Lambda^{(d-2)/2} \sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

• Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

- Either directly: $\partial_t S_{\star}[\varphi] = 0$
- Or from relevant (source-dependent) perturbations

What have we gained?

• Suppose we work with dimensionless variables:

$$x o x/\Lambda, \qquad \phi(x) o \phi(x) \Lambda^{(d-2)/2} \sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

• Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

- Either directly: $\partial_t S_{\star}[\varphi] = 0$
- Or from relevant (source-dependent) perturbations

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● のへで

What have we gained?

Suppose we work with dimensionless variables:

$$x \to x/\Lambda, \qquad \phi(x) \to \phi(x)\Lambda^{(d-2)/2}\sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

• Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

- Either directly: $\partial_t S_{\star}[\varphi] = 0$
- Or from relevant (source-dependent) perturbations

What have we gained?

Suppose we work with dimensionless variables:

$$x \to x/\Lambda, \qquad \phi(x) \to \phi(x)\Lambda^{(d-2)/2}\sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

• Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

Nonperturbatively renormalizable solutions follow from fixed-points

• Either directly: $\partial_t S_{\star}[\varphi] = 0$

• Or from relevant (source-dependent) perturbations

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● のへで

What have we gained?

Suppose we work with dimensionless variables:

$$x \to x/\Lambda, \qquad \phi(x) \to \phi(x)\Lambda^{(d-2)/2}\sqrt{Z}, \qquad t = \ln \mu/\Lambda$$

• Define a fixed-point as a scale-invariant action:

$$\partial_t S_\star[\phi] = 0$$

- Either directly: $\partial_t S_{\star}[\varphi] = 0$
- Or from relevant (source-dependent) perturbations

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Flows in Theory Space

Flows in Theory Space



Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Cextbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- ϕ . Suppose that S_Λ obeys the Polchinski equation
- ϕ . Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_\Lambda$

- To show that there is more to the answer
- . To convince you that the question is profound

ヘロア 人間 アメヨア イヨア

э

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_{\Lambda}$

- To show that there is more to the answer
- To convince you that the question is profound

ヘロト ヘアト ヘリト

э

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_{\Lambda}$

- To show that there is more to the answer
- To convince you that the question is profound

ヘロア 人間 アメヨア イヨア

э

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
 - Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_{\Lambda}$

- To show that there is more to the answer
- To convince you that the question is profound

ヘロア 人間 アメヨア イヨア

ж

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_{\Lambda}$

- To show that there is more to the answer
- To convince you that the question is profound

・ コ マ く 雪 マ く ヨ マ ー ヨ

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_\Lambda$

- To show that there is more to the answer
- To convince you that the question is profound

・ コ マ く 雪 マ く ヨ マ ー ヨ

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_{\Lambda}$

- To show that there is more to the answer
- To convince you that the question is profound

・ コ マ く 雪 マ く ヨ マ ー ヨ

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_\Lambda$

- To show that there is more to the answer
- To convince you that the question is profound
Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_\Lambda$

My aims in this talk

- To show that there is more to the answer
- To convince you that the question is profound

▲口>▲母>▲目>▲目> 目 のみぐ

イロト 不得下 イヨト イヨト

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_\Lambda$

My aims in this talk

- To show that there is more to the answer
- To convince you that the question is profound

Textbook versus Wilsonian

Merits of the latter already appreciated by this audience!

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of the WEA

The Simple Answer

- Suppose that S_{Λ} obeys the Polchinski equation
- Then the correlation functions are generated by $\lim_{\Lambda \to 0} S_{\Lambda}$

My aims in this talk

- To show that there is more to the answer
- To convince you that the question is profound









Formulation

Demand invariance of the partition function under blocking

$\circ \wedge \partial_h Z = \wedge \partial_h \int \mathcal{D}\phi \, e^{-S_h^{(0)}[\phi]} = \int \mathcal{D}\phi \, \frac{1}{2} \phi = 0$

Parametrizes the blocking procedure

Alternative point of view

- Under RG step $\Lambda \to \Lambda \delta \Lambda$, $\phi \to \phi \Psi \delta \Lambda / \Lambda$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda\partial_{\Lambda}e^{-S^{
m tot}_{\Lambda}[\phi]}=\int\!\!d^d\!x\,rac{\partial}{\delta\phi(x)}\left\{\Psi(x)e^{-S^{
m tot}_{\Lambda}[\phi]}
ight\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \ e^{-S_{\Lambda}^{\text{tot}}[\phi]} = \int \mathcal{D}\phi \ \frac{\delta}{\delta \phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda
 ightarrow \Lambda \delta \Lambda, \ \phi
 ightarrow \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^{d}x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^{d}x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S_{\Lambda}^{\text{tot}}[\phi]} = \int d^{d}x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S_{\Lambda}^{\text{tot}}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \ e^{-S_{\Lambda}^{\text{tot}}[\phi]} = \int \mathcal{D}\phi \ \frac{\delta}{\delta \phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda o \Lambda \delta \Lambda, \ \phi o \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^d x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^d x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda\partial_{\Lambda}e^{-S^{
m tot}_{\Lambda}[\phi]}=\int\!d^d\!x\,rac{\delta}{\delta\phi(x)}\left\{\Psi(x)e^{-S^{
m tot}_{\Lambda}[\phi]}
ight\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \ e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \ \frac{\delta}{\delta \phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda o \Lambda \delta \Lambda, \ \phi o \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^d x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^d x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S_{\Lambda}^{\text{tot}}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta \phi} \dots = 0$$

Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda o \Lambda \delta \Lambda, \ \phi o \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^d x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^d x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta\phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda o \Lambda \delta \Lambda, \ \phi o \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^{d}x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^{d}x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta\phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda \rightarrow \Lambda \delta \Lambda$, $\phi \rightarrow \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^{d}x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^{d}x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta\phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda \rightarrow \Lambda \delta \Lambda$, $\phi \rightarrow \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^{d}x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^{d}x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta\phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda \rightarrow \Lambda \delta \Lambda$, $\phi \rightarrow \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^{d}x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^{d}x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta \phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda \rightarrow \Lambda \delta \Lambda$, $\phi \rightarrow \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int d^d x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int d^d x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

Formulation

• Demand invariance of the partition function under blocking

$$-\Lambda \partial_{\Lambda} e^{-S^{\rm tot}_{\Lambda}[\phi]} = \int \! d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-S^{\rm tot}_{\Lambda}[\phi]} \right\}$$

•
$$\Lambda \partial_{\Lambda} \mathcal{Z} = \Lambda \partial_{\Lambda} \int \mathcal{D}\phi \, e^{-S^{\text{tot}}_{\Lambda}[\phi]} = \int \mathcal{D}\phi \, \frac{\delta}{\delta \phi} \cdots = 0$$

• Parametrizes the blocking procedure

Alternative point of view

- Ψ corresponds to a field redefinition
- Under RG step $\Lambda \rightarrow \Lambda \delta \Lambda$, $\phi \rightarrow \phi \Psi \delta \Lambda / \Lambda$

$$-\Lambda \partial_{\Lambda} S^{\text{tot}} = \int \! d^d x \, \frac{\delta S^{\text{tot}}}{\delta \phi(x)} \Psi(x) - \int \! d^d x \, \frac{\delta \Psi(x)}{\delta \phi(x)}$$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > ... □

The Modified Polchinski Equation I

- Introduce UV cutoff function, K(p²/Λ²)
 quasi-local, with K(0) = 1 and K(∞) = 0
- Define regularized propagator $C_{\Lambda}(p^2) = K(p^2/\Lambda^2)/p^2$
- Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2} \int_{\rho} \phi(\rho) C_{\Lambda}^{-1}(\rho^2) \phi(-\rho)$$

Make the split

$$S^{\text{tot}} = \hat{S} + S$$

The Modified Polchinski Equation I

• Introduce UV cutoff function, $K(p^2/\Lambda^2)$

• quasi-local, with K(0) = 1 and $K(\infty) = 0$

• Define regularized propagator $C_{\Lambda}(p^2) = K(p^2/\Lambda^2)/p^2$

• Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2} \int_{p} \phi(p) C_{\Lambda}^{-1}(p^{2}) \phi(-p)$$

• Make the split

$$S^{\text{tot}} = \hat{S} + S$$

The Modified Polchinski Equation I

- Introduce UV cutoff function, K(p²/Λ²)
 quasi-local, with K(0) = 1 and K(∞) = 0
- Define regularized propagator $C_{\Lambda}(p^2) = K(p^2/\Lambda^2)/p^2$
- Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2} \int_{p} \phi(p) C_{\Lambda}^{-1}(p^{2}) \phi(-p)$$

• Make the split

$$S^{\text{tot}} = \hat{S} + S$$

The Modified Polchinski Equation I

- Introduce UV cutoff function, $K(p^2/\Lambda^2)$
 - quasi-local, with K(0)=1 and $K(\infty)=0$
- Define regularized propagator $C_{\Lambda}(p^2) = K(p^2/\Lambda^2)/p^2$

• Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2} \int_{p} \phi(p) C_{\Lambda}^{-1}(p^{2}) \phi(-p)$$

• Make the split

$$S^{\text{tot}} = \hat{S} + S$$

The Modified Polchinski Equation I

- Introduce UV cutoff function, $K(p^2/\Lambda^2)$
 - quasi-local, with K(0)=1 and $K(\infty)=0$
- Define regularized propagator $C_{\Lambda}(p^2) = \mathcal{K}(p^2/\Lambda^2)/p^2$
- Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2} \int_{p} \phi(p) C_{\Lambda}^{-1}(p^2) \phi(-p)$$

• Make the split

 $S^{\mathrm{tot}} = \hat{S} + S$

The Modified Polchinski Equation I

- Introduce UV cutoff function, $K(p^2/\Lambda^2)$
 - quasi-local, with K(0)=1 and $K(\infty)=0$
- Define regularized propagator $C_{\Lambda}(p^2) = K(p^2/\Lambda^2)/p^2$
- Introduce regularized kinetic term:

$$\hat{S} = \frac{1}{2} \int_{p} \phi(p) C_{\Lambda}^{-1}(p^2) \phi(-p)$$

Make the split

$$S^{\text{tot}} = \hat{S} + S$$

The Modified Polchinski Equation II

• Define $\dot{C} \equiv -\Lambda \partial_{\Lambda} C$ and take

$$\Psi(\rho) = \frac{1}{2}\dot{C}_{\Lambda}(\rho^2)\frac{\delta(S^{\text{tot}} - 2\hat{S})}{\delta\phi(\rho)} + \psi(\rho)$$

where ψ allows for an extra field redefinition along the flow The modified Polchinski equation is:

$$-\Lambda\partial_{\Lambda}S = \frac{1}{2}\frac{\delta S}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} - \frac{1}{2}\frac{\delta}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} + \psi \cdot \frac{\delta S^{\text{tot}}}{\delta\phi} - \frac{\delta}{\delta\phi} \cdot \psi$$

ヘロト ヘ部ト ヘヨト ヘヨト

æ

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● のへで

The Modified Polchinski Equation II

• Define $\dot{C} \equiv -\Lambda \partial_{\Lambda} C$ and take

$$\Psi(p) = rac{1}{2}\dot{C}_{\Lambda}(p^2)rac{\delta(S^{ ext{tot}}-2\hat{S})}{\delta\phi(p)} + \psi(p)$$

where ψ allows for an extra field redefinition along the flow \bullet The modified Polchinski equation is:

$$-\Lambda\partial_{\Lambda}S = \frac{1}{2}\frac{\delta S}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} - \frac{1}{2}\frac{\delta}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} + \psi \cdot \frac{\delta S^{\text{tot}}}{\delta\phi} - \frac{\delta}{\delta\phi} \cdot \psi$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

The Modified Polchinski Equation II

• Define $\dot{C} \equiv -\Lambda \partial_{\Lambda} C$ and take

$$\Psi(p) = rac{1}{2}\dot{C}_{\Lambda}(p^2)rac{\delta(S^{ ext{tot}}-2\hat{S})}{\delta\phi(p)} + \psi(p)$$

where ψ allows for an extra field redefinition along the flow \bullet The modified Polchinski equation is:

$$-\Lambda\partial_{\Lambda}S = \frac{1}{2}\frac{\delta S}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} - \frac{1}{2}\frac{\delta}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} + \psi \cdot \frac{\delta S^{\text{tot}}}{\delta\phi} - \frac{\delta}{\delta\phi} \cdot \psi$$

◆□ > ◆□ > ◆臣 > ◆臣 > ─ 臣 ─ のへで

The Modified Polchinski Equation II

• Define $\dot{C} \equiv -\Lambda \partial_{\Lambda} C$ and take

$$\Psi(p) = rac{1}{2}\dot{C}_{\Lambda}(p^2)rac{\delta(S^{ ext{tot}}-2\hat{S})}{\delta\phi(p)} + \psi(p)$$

where ψ allows for an extra field redefinition along the flow

• The modified Polchinski equation is:

$$-\Lambda\partial_{\Lambda}S = \frac{1}{2}\frac{\delta S}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} - \frac{1}{2}\frac{\delta}{\delta\phi} \cdot \dot{C} \cdot \frac{\delta S}{\delta\phi} + \psi \cdot \frac{\delta S^{\text{tot}}}{\delta\phi} - \frac{\delta}{\delta\phi} \cdot \psi$$

▲口 → ▲御 → ▲注 → ▲注 → □注 □

Rescalings

Remove the canonical dimensions

$$\tilde{p} = p/\Lambda, \qquad \varphi(\tilde{p}) = \phi(p) \Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda \frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action
- This process is (essentially) equivalent to $\phi(p) \rightarrow \phi(p) \Lambda^{-(d+2)/2} \sqrt{Z}$

Rescalings

Remove the *canonical* dimensions

$$\tilde{p} = p/\Lambda, \qquad \varphi(\tilde{p}) = \phi(p)\Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda \frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action
- This process is (essentially) equivalent to $\phi(p) \rightarrow \phi(p) \Lambda^{-(d+2)/2} \sqrt{Z}$

Rescalings

• Remove the *canonical* dimensions

$$ilde{p} = p/\Lambda, \qquad arphi(ilde{p}) = \phi(p) \Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda \frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action
- This process is (essentially) equivalent to $\phi(p) \rightarrow \phi(p) \Lambda^{-(d+2)/2} \sqrt{Z}$

▲ロ ▶ ▲周 ▶ ▲ 国 ▶ ▲ 国 ▶ ● の Q @

Rescalings

• Remove the *canonical* dimensions

$$ilde{p} = p/\Lambda, \qquad arphi(ilde{p}) = \phi(p) \Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda \frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action
- This process is (essentially) equivalent to $\phi(p) \rightarrow \phi(p) \Lambda^{-(d+2)/2} \sqrt{Z}$

Rescalings

• Remove the *canonical* dimensions

$$ilde{p} = p/\Lambda, \qquad arphi(ilde{p}) = \phi(p) \Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda \frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action
- This process is (essentially) equivalent to $\phi(p) \rightarrow \phi(p) \Lambda^{-(d+2)/2} \sqrt{Z}$

Rescalings

• Remove the *canonical* dimensions

$$ilde{
ho}=
ho/\Lambda,\qquad arphi(ilde{
ho})=\phi(
ho)\Lambda^{(d+2)/2}$$

(Henceforth drop tildes)

$$\psi = -\frac{1}{2}\eta\varphi, \qquad \eta \equiv \Lambda \frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action
- This process is (essentially) equivalent to $\phi(p) \rightarrow \phi(p) \Lambda^{-(d+2)/2} \sqrt{Z}$

$\left(\partial_t - \hat{D}\right) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$

with

$$\hat{D} = \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)}$$

- Fixed-points follow from $\partial_t S_{\star}[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions followfrom renormalizability of S?

$$(\partial_t - \hat{D}) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot \mathcal{C}^{-1} \cdot \varphi$$

$$\hat{D} = \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)}$$

- Fixed-points follow from $\partial_t S_{\star}[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of *S*?

$$(\partial_t - \hat{D}) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$
with
$$\hat{D} = \int_p \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_p \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)}$$

- Fixed-points follow from $\partial_t S_\star[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of *S*?

$$\left(\partial_t - \hat{D}\right) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

with

$$\hat{D} = \int_{p} \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_{p} \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)}$$

- Fixed-points follow from $\partial_t S_{\star}[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of S?

$$\left(\partial_t - \hat{D}\right) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

with

$$\hat{D} = \int_{p} \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_{p} \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)}$$

- Fixed-points follow from $\partial_t S_\star[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of *S*?
Rescaled flow equation

$$\left(\partial_t - \hat{D}\right) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

with

$$\hat{D} = \int_{p} \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_{p} \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)}$$

- Fixed-points follow from $\partial_t S_\star[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

Renormalizability

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of *S*?

Rescaled flow equation

$$\left(\partial_t - \hat{D}\right) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

with

$$\hat{D} = \int_{p} \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_{p} \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)}$$

- Fixed-points follow from $\partial_t S_\star[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

Renormalizability

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of *S*?

Rescaled flow equation

$$\left(\partial_t - \hat{D}\right) S_t[\varphi] = \frac{\delta S}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta S}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

with

$$\hat{D} = \int_{p} \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_{p} \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)}$$

- Fixed-points follow from $\partial_t S_\star[\varphi] = 0$
- η_{\star} is quantized at critical fixed-points

Renormalizability

- S does not satisfy the plain Polchinski equation
- How does renormalizability of the correlation functions follow from renormalizability of *S*?

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a
 - manifestly gauge invariant flow equation
- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role.

Aside: Gauge Theory

• It is possible to construct a gauge invariant cutoff, using

- Covariant higher derivatives
- Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

・ロト ・ 一下・ ・ ヨト ・ 日 ・

э

Aside: Gauge Theory

It is possible to construct a gauge invariant cutoff, using

- Covariant higher derivatives
- Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

・ロト ・ 理 ト ・ ヨ ト ・ ヨ ト

3

Aside: Gauge Theory

It is possible to construct a gauge invariant cutoff, using

- Covariant higher derivatives
- Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
 - Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
 - Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

▲日▼▲□▼▲□▼▲□▼ □ ののの

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
 - Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
 - Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
 - Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role

Aside: Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
 - Covariant higher derivatives
 - Pauli-Villars fields
- Ψ can be chosen to give a

manifestly gauge invariant flow equation

- No gauge fixing is required at any stage!
- The formalism is very complicated

- We must consider expectation values of manifestly gauge invariant operators
- The standard correlation functions play no role









The Standard Correlation Functions

Introduce a source term in the bare action

$$Z[J] = \int \mathcal{D}\phi \, e^{-S_0[\phi] + J\phi}$$

lpha Extract the connected correlation functions from $W[J]\equiv \ln Z$

$$\frac{\delta}{|J|^{1/2}} = \frac{\delta}{|J|^{1/2}} = \frac{\delta}{|J|^{$$

Composite Operators

- \bullet Add additional source terms e.g. $J_2 \cdot \phi \phi$
- Take derivatives with respect to J and J_2 to find

 $\langle \phi(\mathbf{x}_1) \cdots \phi(\mathbf{x}_n) \phi^2(\mathbf{y}_1) \cdots \phi^2(\mathbf{y}_m) \rangle_{\text{comm}}$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $W[J] \equiv \ln Z$

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle_{\mathrm{conn}} = \frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}W[J]\Big|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

 $\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\mathrm{conn}}$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $W[J] \equiv \ln Z$

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle_{\mathrm{conn}} = \frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}W[J]\Big|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

 $\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\mathrm{conn}}$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $\mathcal{W}[J]\equiv \ln \mathcal{Z}$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \Big|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

 $\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\mathrm{conn}}$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $W[J] \equiv \ln \mathcal{Z}$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \Big|_{J=0}$$

Composite Operators

- Add additional source terms *e.g.* $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

$$\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\text{control}}$$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $W[J] \equiv \ln \mathcal{Z}$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \Big|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

$$\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\text{conn}}$$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $W[J] \equiv \ln \mathcal{Z}$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \Big|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

$$\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\mathrm{conm}}$$

The Standard Correlation Functions

• Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J \cdot \phi}$$

• Extract the connected correlation functions from $W[J] \equiv \ln \mathcal{Z}$

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{conn}} = \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} W[J] \Big|_{J=0}$$

Composite Operators

- Add additional source terms e.g. $J_2 \cdot \phi^2$
- Take derivatives with respect to J and J_2 to find

$$\langle \phi(x_1)\cdots\phi(x_n)\phi^2(y_1)\cdots\phi^2(y_m)\rangle_{\mathrm{conm}}$$

◆□ → ◆□ → ◆注 → ◆注 → □ □ □

New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for J-dependence of the action

 $S_{\Lambda}[\phi] \to T_{\Lambda}[\phi, J]$

• The flow equation follows as before

$$-\Lambda \partial_{\Lambda} e^{-T_{\Lambda}^{\rm tot}[\phi, J]} = \int \! d^d \! x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_{\Lambda}^{\rm tot}[\phi, J]} \right\}$$

• A sensible boundary condition would be

$$\lim_{\Lambda\to\Lambda_0}T_{\Lambda}[\phi,J]-S_{\Lambda}[\phi]=-J\cdot\phi$$

New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for *J*-dependence of the action

 $S_{\Lambda}[\phi] \to T_{\Lambda}[\phi, J]$

• The flow equation follows as before

$$-\Lambda \partial_{\Lambda} e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} = \int d^{d}x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} \right\}$$

• A sensible boundary condition would be

$$\lim_{\Lambda \to \Lambda_0} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for *J*-dependence of the action

 $S_{\Lambda}[\phi] \to T_{\Lambda}[\phi,J]$

• The flow equation follows as before

$$-\Lambda \partial_{\Lambda} e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} = \int d^{d}x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} \right\}$$

• A sensible boundary condition would be

$$\lim_{\Lambda \to \Lambda_0} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for *J*-dependence of the action

 $S_{\Lambda}[\phi] \rightarrow T_{\Lambda}[\phi, J]$

• The flow equation follows as before

$$-\Lambda \partial_{\Lambda} e^{-\mathcal{T}^{\rm tot}_{\Lambda}[\phi,J]} = \int d^d x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-\mathcal{T}^{\rm tot}_{\Lambda}[\phi,J]} \right\}$$

• A sensible boundary condition would be

$$\lim_{\Lambda \to \Lambda_0} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for *J*-dependence of the action

 $S_{\Lambda}[\phi] \rightarrow T_{\Lambda}[\phi, J]$

• The flow equation follows as before

$$-\Lambda \partial_{\Lambda} e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} = \int d^{d}x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} \right\}$$

• A sensible boundary condition would be

$$\lim_{\Lambda \to \Lambda_0} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

New ERG Approach

- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for *J*-dependence of the action

 $S_{\Lambda}[\phi] \rightarrow T_{\Lambda}[\phi, J]$

• The flow equation follows as before

$$-\Lambda \partial_{\Lambda} e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} = \int d^{d}x \, \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_{\Lambda}^{\text{tot}}[\phi, J]} \right\}$$

• A sensible boundary condition would be

$$\lim_{\Lambda \to \Lambda_0} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p) \Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$\begin{split} &\left(\partial_{1}-\partial_{-}-D^{2}\right)T_{1}\left[\partial_{+}A\right] = \frac{\partial T}{\partial \varphi} \left[K^{2} \frac{\partial T}{\partial \varphi} - \frac{\partial}{\partial \varphi} \left[K^{2} \frac{\partial T}{\partial \varphi} - \frac{\partial}{\partial \varphi} e^{-\frac{1}{2}} \frac{\partial}{\partial \varphi} e^{-\frac{1}{2}} e^{-\frac$$

・ロ・ ・ 日・ ・ 田・ ・ 田・

э

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 ● のへで

Rescalings again

• Treat ϕ as before

• Introduce the dimensionless source

$$j(\tilde{p}) = j(p) \Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$\left(\partial_t - \hat{D} - \hat{D}^J\right) \mathcal{T}_t[\varphi, J] = \frac{\delta T}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

$$\hat{D} = \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)}$$

$$\hat{D}^J = \int_{\rho} \left[\left(d - d_J + \rho \cdot \partial_{\rho} \right) J(\rho) \right] \frac{\delta}{\delta J(\rho)}$$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p)\Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$(\partial_t - \hat{D} - \hat{D}^J) T_t[\varphi, J] = \frac{\delta T}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

• $\hat{D} = \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)}$
• $\hat{D}^J = \int_{\rho} \left[\left(d - d_J + \rho \cdot \partial_{\rho} \right) J(\rho) \right] \frac{\delta}{\delta J(\rho)}$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p)\Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$\left(\partial_t - \hat{D} - \hat{D}^J\right) T_t[\varphi, J] = \frac{\delta T}{\delta \varphi} \cdot K' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot K' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot C^{-1} \cdot \varphi$$

• $\hat{D} = \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)}$
• $\hat{D}^J = \int_{\rho} \left[\left(d - d_J + \rho \cdot \partial_{\rho} \right) J(\rho) \right] \frac{\delta}{\delta J(\rho)}$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p) \Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$(\partial_{t} - \hat{D} - \hat{D}^{J}) T_{t}[\varphi, J] = \frac{\delta T}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot \mathcal{C}^{-1} \cdot \varphi$$

$$\hat{D} = \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)}$$

$$\hat{D}^{J} = \int_{\rho} \left[\left(d - d_{J} + \rho \cdot \partial_{\rho} \right) J(\rho) \right] \frac{\delta}{\delta J(\rho)}$$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p) \Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$\begin{aligned} \left(\partial_t - \hat{D} - \hat{D}^J\right) T_t[\varphi, J] &= \frac{\delta T}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot \mathcal{C}^{-1} \cdot \varphi \\ \bullet \quad \hat{D} &= \int_p \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_p \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)} \\ \bullet \quad \hat{D}^J &= \int_p \left[\left(d - d_J + p \cdot \partial_p \right) J(p) \right] \frac{\delta}{\delta J(p)} \end{aligned}$$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p)\Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$\begin{aligned} \left(\partial_{t} - \hat{D} - \hat{D}^{J}\right) T_{t}[\varphi, J] &= \frac{\delta T}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot \mathcal{C}^{-1} \cdot \varphi \\ \bullet \quad \hat{D} &= \int_{\rho} \left[\left(\frac{d+2-\eta}{2} + \rho \cdot \partial_{\rho} \right) \varphi(\rho) \right] \frac{\delta}{\delta \varphi(\rho)} \\ \bullet \quad \hat{D}^{J} &= \int_{\rho} \left[\left(d - d_{J} + \rho \cdot \partial_{\rho} \right) J(\rho) \right] \frac{\delta}{\delta J(\rho)} \end{aligned}$$

Rescalings again

- Treat ϕ as before
- Introduce the dimensionless source

$$j(\tilde{p}) = j(p) \Lambda^{d-d_J}, \qquad \tilde{p} = p/\Lambda$$

• This gives the source-dependent flow equation

$$\begin{aligned} \left(\partial_t - \hat{D} - \hat{D}^J\right) T_t[\varphi, J] &= \frac{\delta T}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\delta}{\delta \varphi} \cdot \mathcal{K}' \cdot \frac{\delta T}{\delta \varphi} - \frac{\eta}{2} \varphi \cdot \mathcal{C}^{-1} \cdot \varphi \\ \bullet \quad \hat{D} &= \int_p \left[\left(\frac{d+2-\eta}{2} + p \cdot \partial_p \right) \varphi(p) \right] \frac{\delta}{\delta \varphi(p)} \\ \bullet \quad \hat{D}^J &= \int_p \left[\left(d - d_J + p \cdot \partial_p \right) J(p) \right] \frac{\delta}{\delta J(p)} \end{aligned}$$
・ロト ・雪 ト ・ 画 ト ・ 画 ト

э

Source-Dependent Renormalization

- Either directly: $\partial_t T_*[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

Source-Dependent Renormalization

- Either directly: $\partial_t T_*[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

Source-Dependent Renormalization

- Either directly: $\partial_t T_{\star}[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

Source-Dependent Renormalization

- Either directly: $\partial_t T_{\star}[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

Suppose that we have found a critical fixed-point

 $\partial_t S_\star[\varphi] = 0$

Then there is always a source-dependent f-p

 $\begin{aligned} \mathcal{T}_{\star}[\varphi, j] &= S_{\star}[\varphi] + \left[e^{-\tilde{j} \cdot \varrho \cdot \delta/\delta \varphi} - 1 \right] \left[S_{\star}[\varphi] + \frac{1}{2} \varphi \cdot C^{-1} \left(1 + \varrho^{-1} \right) \cdot \varphi \right] \\ &= \tilde{j}(\rho) = j(\rho)/\rho^{2} \\ &= \rho(\rho^{2}) = \rho^{2} + O\left(\rho^{4}\right) \text{ (quasi-local)} \end{aligned}$

- The solution only works if $d_2=(d+2-\eta_4)/2$
- In dimensionful variables

$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi,J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$T_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1\right] \left[S_{\star}[\varphi] + \frac{1}{2}\varphi \cdot C^{-1}(1 + \varrho^{-1}) \cdot \varphi\right]$$

• $\overline{j}(p) \equiv j(p)/p^{2}$
• $\varrho = \varrho(p^{2}) = p^{2} + O(p^{4}) \text{ (quasi-local)}$

- The solution only works if $d_J = (d + 2 \eta_{\star})/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$T_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1 \right] \left[S_{\star}[\varphi] + \frac{1}{2} \varphi \cdot C^{-1} \left(1 + \varrho^{-1} \right) \cdot \varphi \right]$$

• $\overline{j}(p) \equiv j(p)/p^{2}$
• $\varrho = \varrho(p^{2}) = p^{2} + O(p^{4}) \text{ (quasi-local)}$

- The solution only works if $d_J = (d+2-\eta_\star)/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$T_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1\right] \left[S_{\star}[\varphi] + \frac{1}{2}\varphi \cdot C^{-1}(1 + \varrho^{-1}) \cdot \varphi\right]$$

• $\overline{j}(\rho) \equiv j(\rho)/\rho^{2}$
• $\varrho = \varrho(\rho^{2}) = \rho^{2} + O(\rho^{4}) \text{ (quasi-local)}$

- The solution only works if $d_J = (d + 2 \eta_*)/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$T_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1\right] \left[S_{\star}[\varphi] + \frac{1}{2}\varphi \cdot C^{-1}\left(1 + \varrho^{-1}\right) \cdot \varphi\right]$$

• $\overline{j}(\rho) \equiv j(\rho)/\rho^{2}$
• $\varrho = \varrho(\rho^{2}) = \rho^{2} + O\left(\rho^{4}\right)$ (quasi-local)

- The solution only works if $d_J = (d + 2 \eta_{\star})/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$T_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1 \right] \left[S_{\star}[\varphi] + \frac{1}{2} \varphi \cdot C^{-1} \left(1 + \varrho^{-1} \right) \cdot \varphi \right]$$

• $\overline{j}(p) \equiv j(p)/p^2$
• $\varrho = \varrho(p^2) = p^2 + O(p^4)$ (quasi-local)

- The solution only works if $d_J = (d + 2 \eta_*)/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$T_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1 \right] \left[S_{\star}[\varphi] + \frac{1}{2} \varphi \cdot C^{-1} \left(1 + \varrho^{-1} \right) \cdot \varphi \right]$$

• $\overline{j}(p) \equiv j(p)/p^2$
• $\varrho = \varrho(p^2) = p^2 + O(p^4)$ (quasi-local)

- The solution only works if $d_J = (d + 2 \eta_\star)/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

• Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

• Then there is always a source-dependent f-p

$$\begin{split} & \mathcal{F}_{\star}[\varphi, j] = S_{\star}[\varphi] + \left[e^{-\overline{j} \cdot \varrho \cdot \delta/\delta\varphi} - 1 \right] \left[S_{\star}[\varphi] + \frac{1}{2} \varphi \cdot C^{-1} \left(1 + \varrho^{-1} \right) \cdot \varphi \right] \\ & \bullet \ \overline{j}(p) \equiv j(p)/p^{2} \\ & \bullet \ \varrho = \varrho(p^{2}) = p^{2} + \mathcal{O}\left(p^{4}\right) \text{ (quasi-local)} \end{split}$$

- The solution only works if $d_J = (d + 2 \eta_\star)/2$
- In dimensionful variables

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

- $S_{\star}[\varphi] = 0$, with $\eta_{\star} = 0$
- **Dimensionless Variables**
 - $= \frac{1}{p} \left[\frac{1}{p} \left(\overline{p} \right) \frac{1}{p} \left(\overline{p} \right) + \frac{1}{2} \int_{\overline{p}} f(\overline{p}) f(-\overline{p}) \frac{h(\overline{p}^2) h(\overline{p}^2)}{p^2} \right]$
 - T_{π} is quasi-local since

$$K(p^2) = 1 + O(p^2)$$

- $V_{i}(p) = -J_{i}(p) + \frac{1}{2} \int J_{i}(p) J_{i}(-p) \frac{K(p^{2}/h^{2}) 1}{p^{2}} dp + \frac{1}{2} \int J_{i}(p) \frac{J_{i}(p) J_{i}(-p) J_{i}(-p) 1}{p^{2}} dp + \frac{1}{2} \int J_{i}(p) \frac{J_{i}(p) J_{i}(-p) 1}{p^{2}} dp + \frac{1}{2} \int J_{i}(p) \frac{J_{i}(p) J_{i}(-p) J_{i}(-p) 1}{p^{2}} dp + \frac{1}{2} \int J_{i}(p) \frac{J_{i}(p) J_{i}(-p) 1}{p^{2}} dp + \frac{1}{2} \int J_{i}(p) \frac{J_{i}(p)$
- $= \lim_{n \to \infty} T_{\Lambda}[0, \mathcal{A}] = -\frac{1}{2} \int_{\mathcal{A}} \mathcal{A}(\rho) \mathcal{A}(-\rho) \frac{1}{2}$

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{\rho}} j(\tilde{\rho}) j(-\tilde{\rho}) \frac{K(\tilde{\rho}^2) - 1}{\tilde{\rho}^2}$$

• T_{\star} is quasi-local since

$$K(\tilde{p}^2) = 1 + \mathcal{O}\left(\tilde{p}^2\right)$$

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{K(\rho^2/\Lambda^2) - 1}{\rho^2}$$

•
$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$$

•
$$\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{1}{\rho^2}$$

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{\rho}} j(\tilde{\rho}) j(-\tilde{\rho}) \frac{K(\tilde{\rho}^2) - 1}{\tilde{\rho}^2}$$

• T_{\star} is quasi-local since

$$K(\tilde{p}^2) = 1 + O(\tilde{p}^2)$$

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{K(\rho^2/\Lambda^2) - 1}{\rho^2}$$

•
$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$$

•
$$\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{1}{\rho^2}$$

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{\rho}} j(\tilde{\rho}) j(-\tilde{\rho}) \frac{K(\tilde{\rho}^2) - 1}{\tilde{\rho}^2}$$

• T_{\star} is quasi-local since

$$K(\tilde{p}^2) = 1 + \mathcal{O}\left(\tilde{p}^2\right)$$

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{K(\rho^2 / \Lambda^2) - 1}{\rho^2}$$

•
$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$$

•
$$\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{1}{\rho^2}$$

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{p}} j(\tilde{p}) j(-\tilde{p}) \frac{K(\tilde{p}^2) - 1}{\tilde{p}^2}$$

• T_{\star} is quasi-local since

$$\mathcal{K}(ilde{p}^2) = 1 + \mathrm{O}\left(ilde{p}^2\right)$$

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{-\pi} J(p) J(-p) \frac{K(p^2/\Lambda^2) - 1}{p^2}$$

•
$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$$

•
$$\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{\rho} J(\rho) J(-\rho) \frac{1}{\rho^2}$$

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{p}} j(\tilde{p}) j(-\tilde{p}) \frac{K(\tilde{p}^2) - 1}{\tilde{p}^2}$$

• T_{\star} is quasi-local since

$$\mathcal{K}(\tilde{p}^2) = 1 + O\left(\tilde{p}^2\right)$$

Dimensionful Variables

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{p} J(p) J(-p) \frac{K(p^{2}/\Lambda^{2}) - 1}{p^{2}}$$

• $\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$
• $\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{p} J(p) J(-p) \frac{1}{p^{2}}$

) 2 ()

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{p}} j(\tilde{p}) j(-\tilde{p}) \frac{K(\tilde{p}^2) - 1}{\tilde{p}^2}$$

• T_{\star} is quasi-local since

$$\mathcal{K}(\tilde{p}^2) = 1 + O\left(\tilde{p}^2\right)$$

Dimensionful Variables

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{p} J(p) J(-p) \frac{K(p^{2}/\Lambda^{2}) - 1}{p^{2}}$$

• $\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$
• $\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{p} J(p) J(-p) \frac{1}{p^{2}}$

) 9 ()

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{p}} j(\tilde{p}) j(-\tilde{p}) \frac{K(\tilde{p}^2) - 1}{\tilde{p}^2}$$

• T_{\star} is quasi-local since

$$\mathcal{K}(\tilde{p}^2) = 1 + O\left(\tilde{p}^2\right)$$

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{p} J(p) J(-p) \frac{K(p^2/\Lambda^2) - 1}{p^2}$$

•
$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$$

•
$$\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{p} J(p) J(-p) \frac{1}{p^2}$$

$$S_\star[arphi]=$$
 0, with $\eta_\star=$ 0

Dimensionless Variables

•
$$T_{\star}[\varphi, j] = -j \cdot \varphi + \frac{1}{2} \int_{\tilde{p}} j(\tilde{p}) j(-\tilde{p}) \frac{K(\tilde{p}^2) - 1}{\tilde{p}^2}$$

• T_{\star} is quasi-local since

$$\mathcal{K}(\tilde{p}^2) = 1 + O\left(\tilde{p}^2\right)$$

•
$$T_{\Lambda}[\phi, J] = -J \cdot \phi + \frac{1}{2} \int_{p} J(p) J(-p) \frac{K(p^2/\Lambda^2) - 1}{p^2}$$

•
$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi, J] = -J \cdot \phi$$

•
$$\lim_{\Lambda \to 0} T_{\Lambda}[0, J] = -\frac{1}{2} \int_{p} J(p) J(-p) \frac{1}{p^2}$$

▲口 → ▲御 → ▲注 → ▲注 → □注 □

And more...

• For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

• Every eigenperturbation, \mathcal{O}_i has a source-dependent extension $\tilde{\mathcal{O}}_i[\varphi,j] = e^{\tilde{j}\cdot \varrho\cdot\delta/\delta\varphi}\mathcal{O}_i$

• At the linear level

$$\mathcal{T}_t[arphi,j] = \mathcal{T}_\star[arphi,j] + \sum_j lpha_i e^{ar{\lambda}_i t} ilde{\mathcal{O}}_i[arphi,j]$$

where $ar{\lambda}_i=\lambda_i$

• For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

• Every eigenperturbation, \mathcal{O}_i has a source-dependent extension

$$ilde{\mathcal{O}}_i[arphi,j] = e^{ar{j}\cdotarrho\cdot\delta/\deltaarphi}\mathcal{O}_i$$

• At the linear level

$$T_t[\varphi,j] = T_\star[\varphi,j] + \sum_i \alpha_i e^{\tilde{\lambda}_i t} \tilde{\mathcal{O}}_i[\varphi,j]$$

where $ilde{\lambda}_i = \lambda_i$

◆□ > ◆□ > ◆ □ > ◆ □ > □ = の < @

• For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

• Every eigenperturbation, \mathcal{O}_i has a source-dependent extension

$$ilde{\mathcal{O}}_i[arphi, j] = e^{\overline{j} \cdot arrho \cdot \delta / \delta arphi} \mathcal{O}_i$$

• At the linear level

$$T_t[\varphi,j] = T_\star[\varphi,j] + \sum_i \alpha_i e^{\tilde{\lambda}_i t} \tilde{\mathcal{O}}_i[\varphi,j]$$

where $ilde{\lambda}_i = \lambda_i$

◆□ > ◆□ > ◆ □ > ◆ □ > □ = の < @

• For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

• Every eigenperturbation, \mathcal{O}_i has a source-dependent extension

$$ilde{\mathcal{O}}_i[arphi, j] = e^{\overline{j} \cdot \varrho \cdot \delta / \delta arphi} \mathcal{O}_i$$

At the linear level

$$T_t[\varphi,j] = T_\star[\varphi,j] + \sum_i \alpha_i e^{\tilde{\lambda}_i t} \tilde{\mathcal{O}}_i[\varphi,j]$$

where $\tilde{\lambda}_i = \lambda_i$

◆□ > ◆□ > ◆ □ > ◆ □ > □ = の < ⊙

• For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

• Every eigenperturbation, \mathcal{O}_i has a source-dependent extension

$$ilde{\mathcal{O}}_i[arphi, j] = e^{\overline{j} \cdot \varrho \cdot \delta / \delta arphi} \mathcal{O}_i$$

At the linear level

$$T_t[\varphi, j] = T_\star[\varphi, j] + \sum_i \alpha_i e^{\tilde{\lambda}_i t} \tilde{\mathcal{O}}_i[\varphi, j]$$

where $\tilde{\lambda}_i = \lambda_i$

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda\to\infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusion

If we use the modified Polchinski equation with $\psi=-\eta \rho/2$

・ロト ・ 同ト ・ ヨト ・ ヨト

э

Interpretation

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusion

If we use the modified Polchinski equation with $\psi=-\eta arphi/2$

・ロト ・ 一下・ ・ ヨト

э

Interpretation

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusior

If we use the modified Polchinski equation with $\psi=-\eta arphi/2$

・ロト ・ 理 ト ・ ヨ ト ・ ヨ ト

э

Interpretation

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusior

If we use the modified Polchinski equation with $\psi=-\eta arphi/2$

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusion

If we use the modified Polchinski equation with $\psi=-\etaarphi/2$

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusion

If we use the modified Polchinski equation with $\psi=-\eta arphi/2$

- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

$$\lim_{\Lambda \to \infty} T_{\Lambda}[\phi, J] - S_{\Lambda}[\phi] = -J \cdot \phi$$

Conclusion

If we use the modified Polchinski equation with $\psi=-\eta arphi/2$

A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

Philosophy

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

A new perspective on QFT?

• Decide which correlation functions to compute

- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

Philosophy

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

Philosophy

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute
- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

・ロト ・ 理 ト ・ ヨ ト ・ ヨ ・

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

▲日▼ ▲□▼ ▲ □▼ ▲ □▼ ■ ● ● ●

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

▲日▼ ▲□▼ ▲ □▼ ▲ □▼ ■ ● ● ●

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

▲日▼ ▲□▼ ▲ □▼ ▲ □▼ ■ ● ● ●

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions

- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

Questions

Modified Polchinski Equation $\psi=-\eta arphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
- Can a link be made with methods of CET?

Other flow equations

- What happens for other flow equations?
- What does this imply for gauge theories?

◆□ ▶ ◆□ ▶ ◆□ ▶ ◆□ ▶ ◆□ ▶

▲口 → ▲御 → ▲注 → ▲注 → □注 □

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

• What other renormalizable source-dependent solutions exist?

- How does the OPE play a role?
- Can a link be made with methods of CFT?

- What happens for other flow equations?
- What does this imply for gauge theories?

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

• What other renormalizable source-dependent solutions exist?

- How does the OPE play a role?
- Can a link be made with methods of CFT?

Other flow equations

- What happens for other flow equations?
- What does this imply for gauge theories?

▲口▼ ▲□▼ ▲目▼ ▲目▼ ▲□ ● ④ ●

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
- Can a link be made with methods of CFT?

- What happens for other flow equations?
- What does this imply for gauge theories?

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
- Can a link be made with methods of CFT?

- What happens for other flow equations?
- What does this imply for gauge theories?

▲日▼ ▲□▼ ▲ □▼ ▲ □▼ ■ ● ● ●

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
- Can a link be made with methods of CFT?

- What happens for other flow equations?
- What does this imply for gauge theories?

▲日▼ ▲□▼ ▲ □▼ ▲ □▼ ■ ● ● ●

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
- Can a link be made with methods of CFT?

- What happens for other flow equations?
- What does this imply for gauge theories?

▲日▼ ▲□▼ ▲ □▼ ▲ □▼ ■ ● ● ●

Questions

Modified Polchinski Equation $\psi = -\eta \varphi/2$

- What other renormalizable source-dependent solutions exist?
- How does the OPE play a role?
- Can a link be made with methods of CFT?

- What happens for other flow equations?
- What does this imply for gauge theories?

◆□ > ◆□ > ◆豆 > ◆豆 > ̄豆 = つへぐ

Ask not what quantum field theory can compute for you, but what you can compute for quantum field theory

Thank you for listening